NATIONAL MAGLAB

GORDON AND BETTY MOORE FOUNDATION

The 8th Maglab Theory Winter School
"Quantum Matter Without Quasiparticles"
6-10 January 2020, Tallahassee, Florida

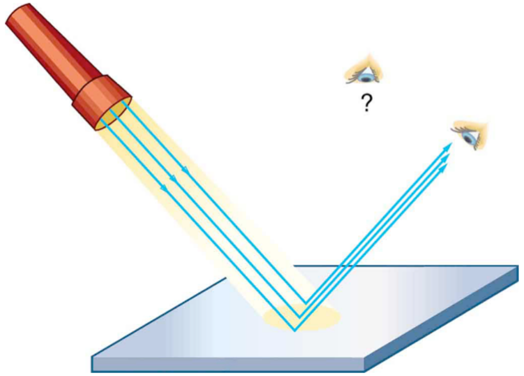
Anushya Chandran (BU)
Liang Fu (MIT)
Hae-Young Kee (U of Toronto)
Philip Kim (Harvard)
Sung-Sik Lee (McMaster & Perimeter)
Leonid Levitov (MIT)
Andrew Lucas (Stanford)
Sri Raghi (Stanford)
Oscar Vafek (FSU & NHMFL)
Cenke Xu (UCSB)

Organizers: Piers Coleman, Rahul Nandkishore, Yuxuan Wang & DM

Transport Properties of Correlated Electron Systems

Dmitrii L. Maslov

Fermi/non-Fermi liquid



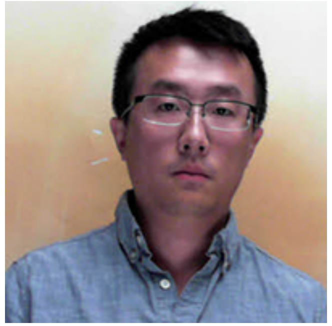
ASCES Summer School
University of Minnesota
10-15 June 2019

Hridis Pal
(PhD, UF→Houston)

Andrey Chubukov
FTPI, U of Minnesota

Vladimir Yudson
Inst. for Optics and Spectroscopy
RAN, Russia

Kamran Behnia
ECPSI

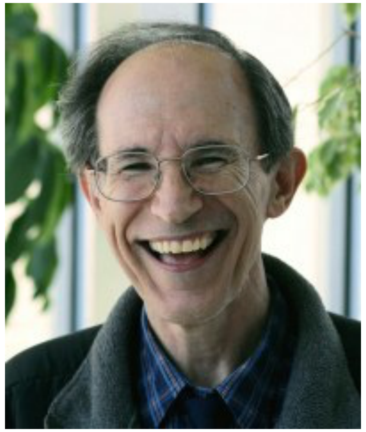


Songci Li
Dirac Fellow
NHMFL

Abhishek Kumar
UF

Catherine Pépin
CEA

Indranil Paul
Paris-Diderot



Gilbert Lonzarich
Cambridge

Sources

1. V. F. Gantmakher and Y. B. Levinson, *Carrier Scattering in Metals and Semiconductors*, Elsevier, 1987
2. E. M. Lifshitz ad L. P. Pitaevskii, *Physical Kinetics,* Course of Theoretical Physics, v. X
(Boltzmann equation, Keldysh technique)
3. G. D. Mahan, Many-Particle Physics, Kluwer Academic (Kubo formula)
4. B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems, Elsevier, 1985*
(quantum corrections)
5. J. Rammer and H. Smith, *Rev. Mod. Phys.* **58**, 323 (1986) (Keldysh technique)
6. H. Smith and H. H. Jensen, *Transport Phenomena, Clarendon, 1989* (transport in Fermi liquids)
7. M. Brando, D. Belitz, F. M., Grosche, and T. R. Kirkpatrick, *Rev. Mod. Phys.* ***88,*** 025006 (2016)
(transport in quantum-critical ferromagnets)
8. H. V. Löhneysen, A. Rosch, M. Vojta},. and P. Wölfle, *Rev. Mod. Phys.* ***79,*** 1015 (2007)
(transport in quantum-critical ferro- and antiferromagnets)
9. D. N. Basov, R. D. Averitt, D. van der Marel, M. Dressel, K. Haule, *Rev. Mod. Phys.* **83**, 471 (2011)
(optical conductivity)
10. R. N. Gurzhi, *Phys. Usp*. **11,** 255 (1968) (hydrodynamic transport in solids)
11. H. Pal, V. I. Yudson, and DM, *Lith. J. Phys*. **52**, 142 (2012)
(conservation laws, umklapps, transport near nematic quantum critical point)
12. A. V. Chubukov and DM, *Rep. Prog. Phys*. **82**, 026503 (2017)
(optical conductivity: 2-band metals, semimetals, SDW QCP)

One of the most serious challenges in Condensed Matter Physics:

A large number of *conducting* compounds do not conform to the predictions of the Fermi-liquid theory

Transport data are perhaps the most extensive and abundant
(*dc*, magnetoresistance, Hall, optics)

…but are notoriously difficult to interpret

Additional conservation laws (momentum and velocity)

The result is usually not an intrinsic property of a Fermi liquid
but depends on coupling to external degrees of freedom (impurities, phonons)

Outline

1. Coherent transport of coherent quasiparticles

1a. Good vs bad (metals)

1b. Drude model and its pitfalls

1c. Resistivity from e-e interaction:

umklapp scattering

normal scattering in i) multiband, ii) compensated, iii)anisotropic metals

1d Optical conductivity

1e The puzzle of charge transport in STO

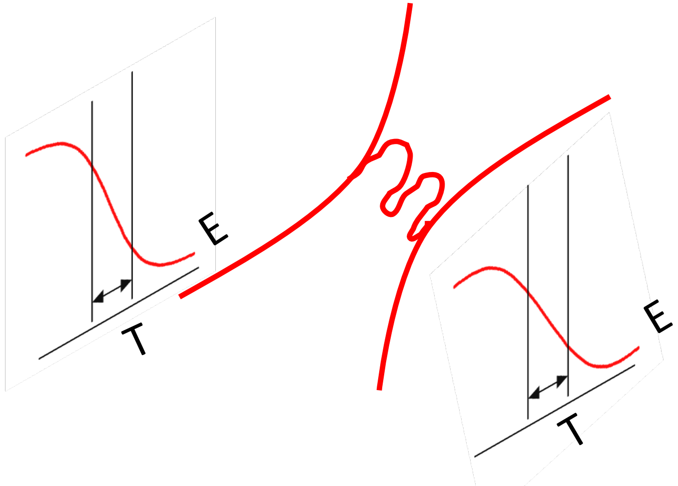
2. Coherent transport of incoherent quasiparticles

2a. Charge and thermal transport near a ferromagnetic quantum critical point

2b. Which mass enters the conductivity?

Fermi-liquid theory: Landau, 1956; 1958

Scattering rate in a weakly interacting Fermi gas



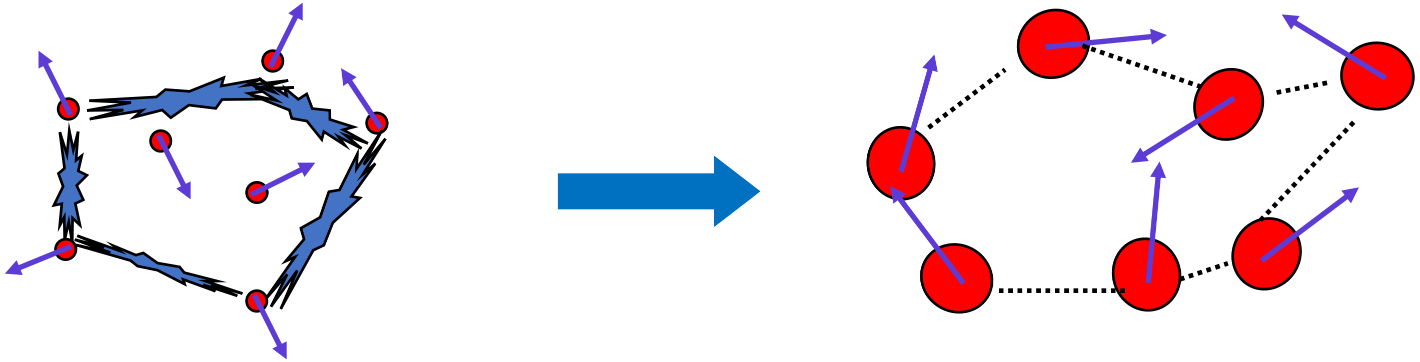
Landau & Pomeranchuk 1936
Baber 1937

$$\frac{1}{\tau} = g\frac{E_F}{\hbar}\left(\frac{k_B T}{E_F}\right)^2$$

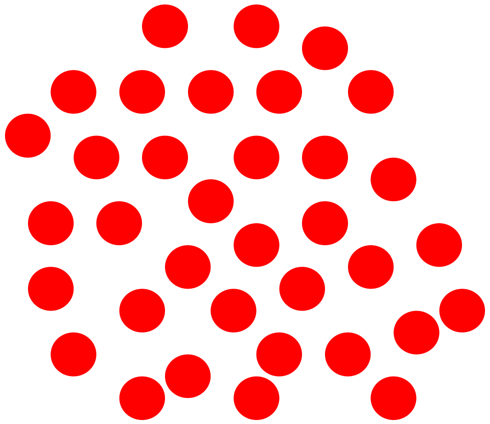
Not an outcome but rather an input of the Fermi-liquid theory

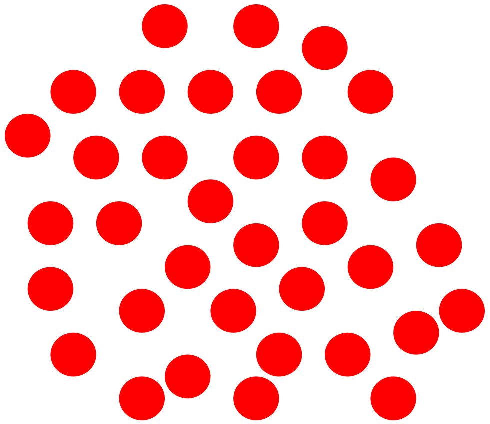
Quantum statistics: At T→0, quasiparticles are free!

Particles (*m,g=2*)→ quasiparticles (*m*, g)**



Fermi Liquid: He3





$\eta \sim \rho_m v_F \ell \propto 1/T^2$

T>E

$T>E_F$:
mean free path~interatomic distance

$T<E_F$:
mean free path>>interatomic distance

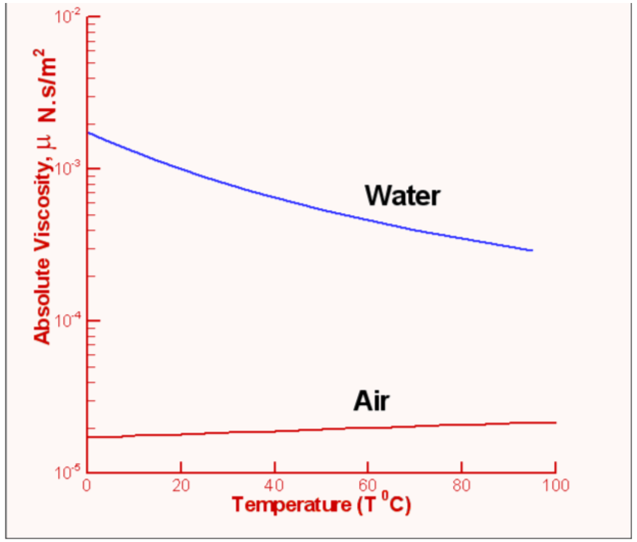
Viscosity ~ classical liquid

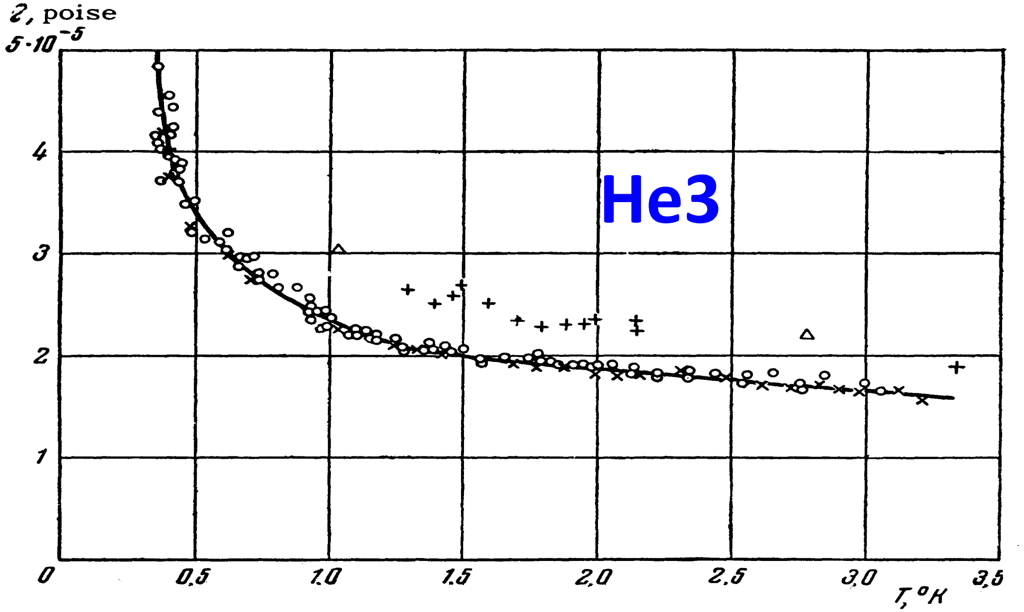
Viscosity ~ dilute gas

$$\eta \propto \exp(\Delta / T)$$

$$\eta \sim \rho v_F \ell \propto T^{-2}$$

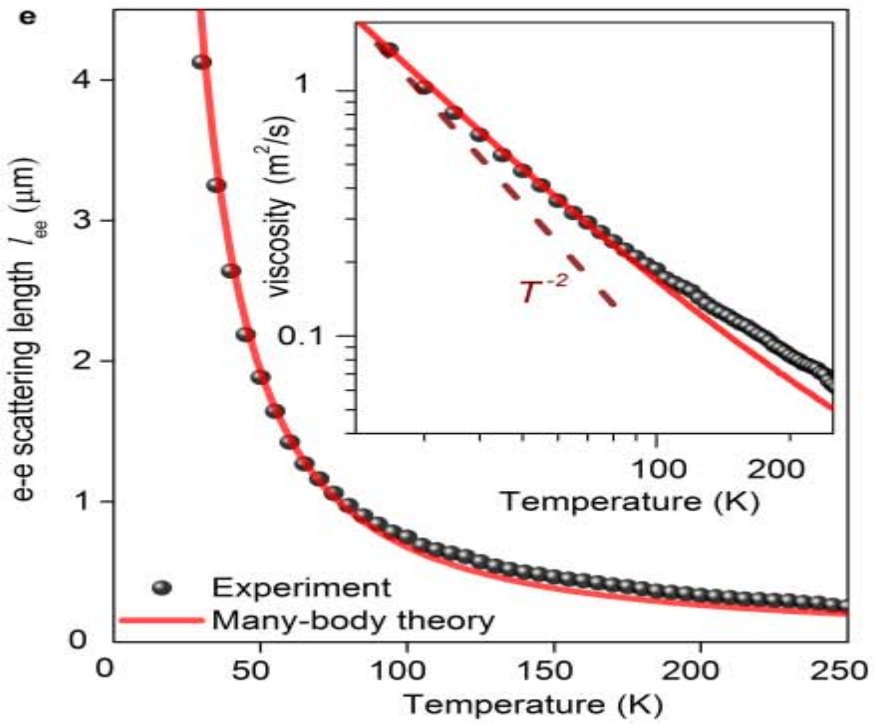
Quantum Fermi Liquid = dilute gas!





Zinov'eva JETP 1958

Graphene

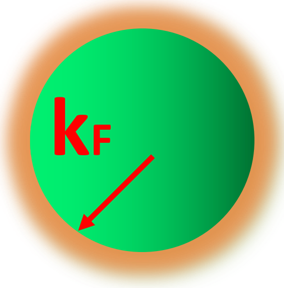


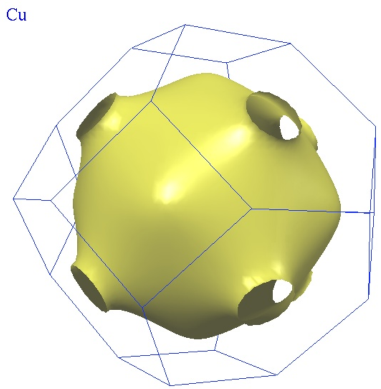
Krishna Kumar et al.

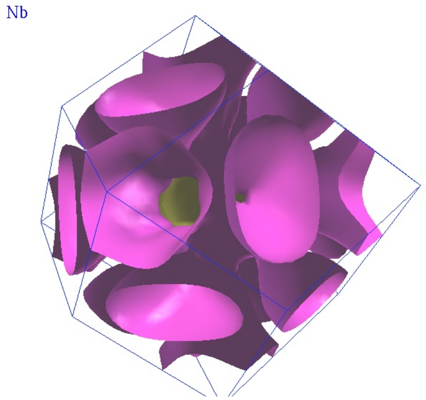
Nature Phys. 2017

Fermi liquid theory:
Galilean-invariant system
of neutral fermions (He^3, cold atoms)

Electrons in solids:
Non-Galilean-invariant system
of charged particles







“anisotropic Fermi liquid”

Original FL theory: quasi-particles are free.
The predictions are only for thermodynamic properties.

Galilean-invariant FL (He^3, neutron stars): Galilean $\otimes$SU(2)$\otimes$U(1)…$\rightarrow$

$$\gamma = \frac{C_V}{T} = \left(\frac{C_V}{T}\right)_0 \frac{m^*}{m_0}, \chi_s = \chi_{s0}\frac{g^*}{2}\frac{m^*}{m_0}$$

Anisotropic FL (metals): U(1)$\otimes$ crystal group

$$\gamma = \frac{C_V}{T} = \gamma_{\text{band}} \frac{m^*}{m_{\text{band}}}$$

$$\chi_s = \chi_{s,\text{band}} \frac{m^*}{m_{\text{band}}} \frac{g^*}{g_{\text{band}}}$$

Hard to quantify

Transport. "Hallmark of the FL behavior: $\rho \propto T^2$."

Outline

1. Coherent transport of coherent quasiparticles

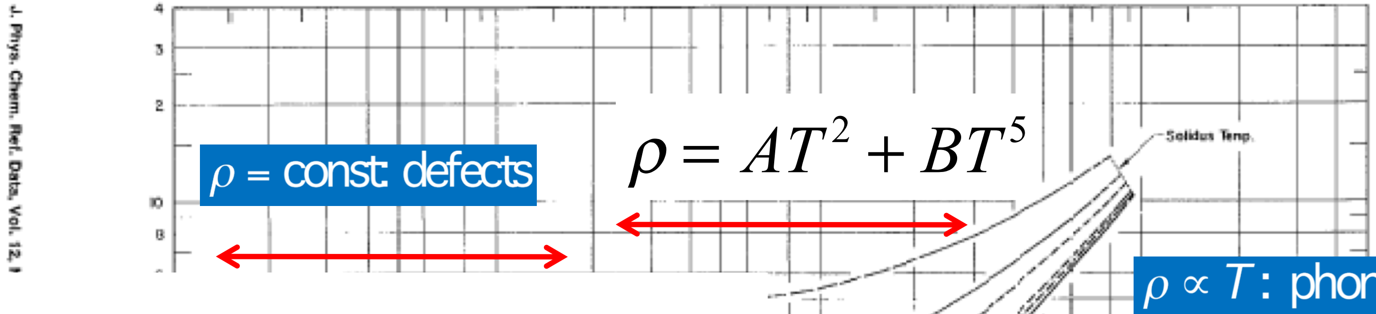
1a. Good vs bad (metals)

Conventional (FL) metals

ρ = const: defects

$\rho = AT^2 + BT^5$

$\rho \propto T$: phonons above the Debye temperature



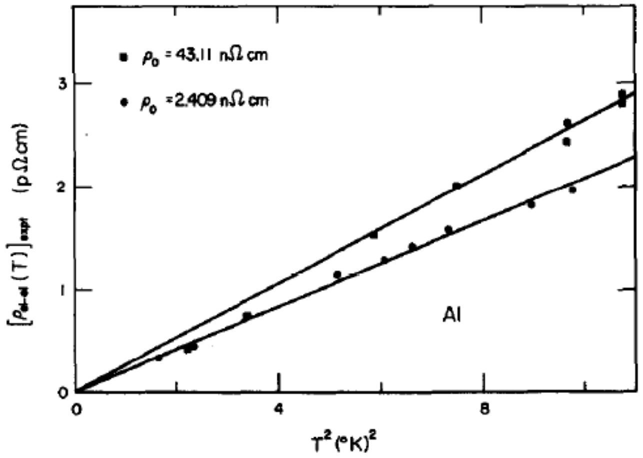


Fig. 1. Plots of $[\rho_{el-el}(T)]_{expt}$ for Al as a function of T^2 for two values of ρ_0. The straight lines through the origin are fitted to the data points.

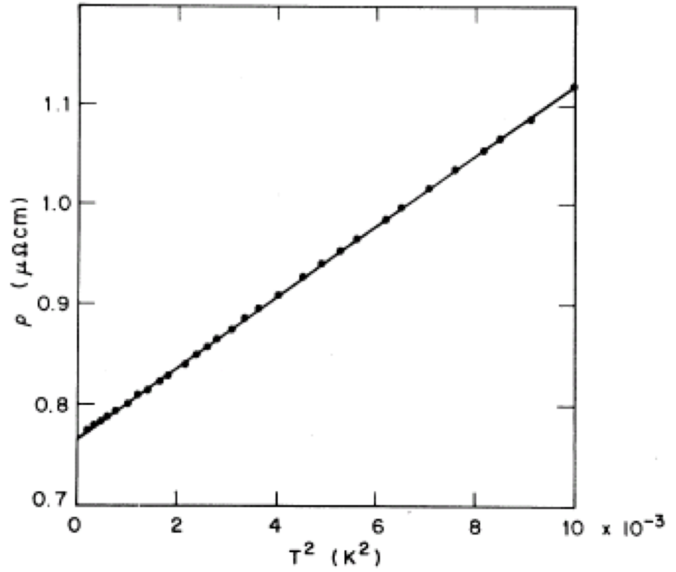
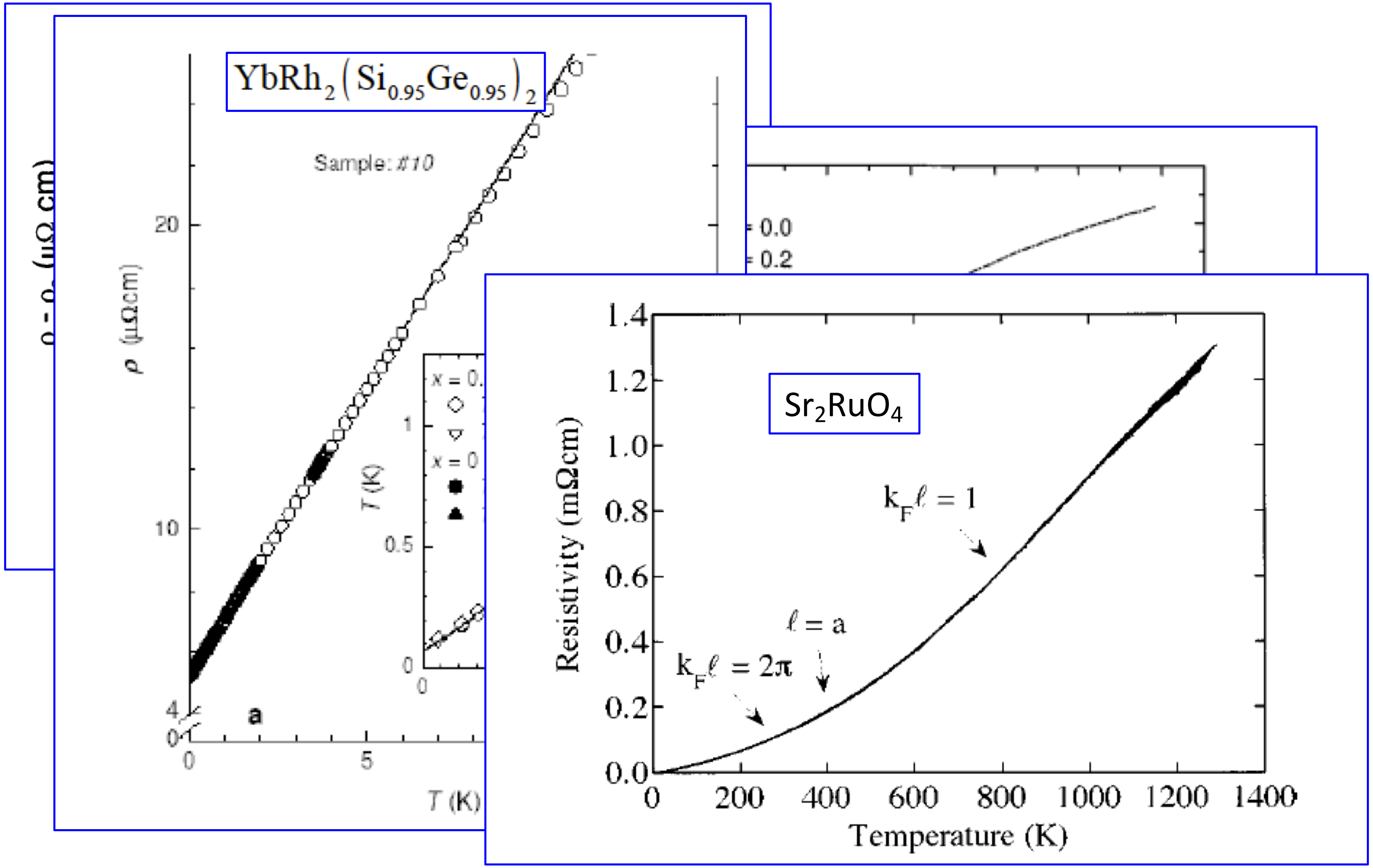
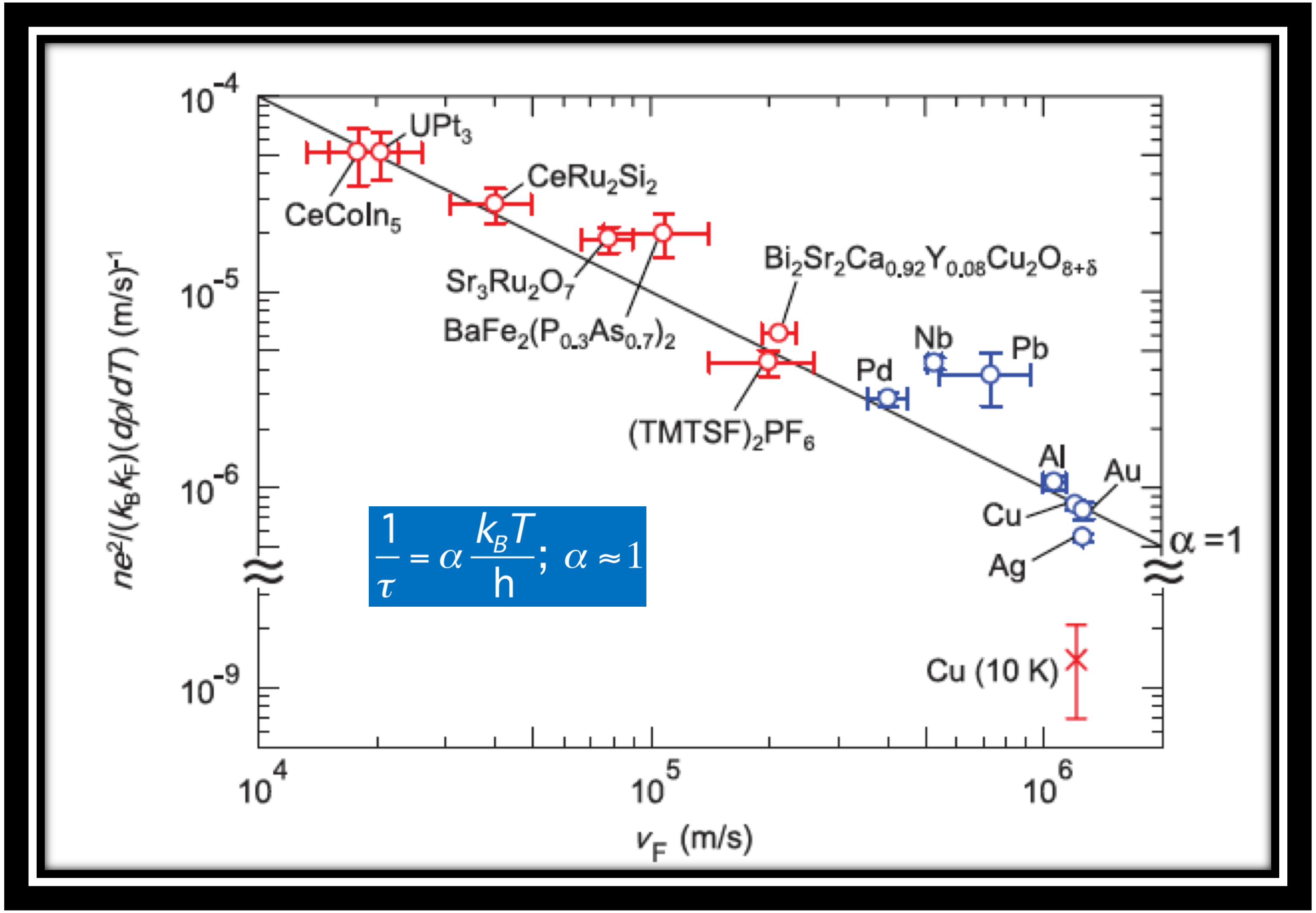


FIG. 3. Electrical resistivity of $CeAl_3$ below 100 mK, plotted against T^2.

Unconventional (“bad”, “strange”, “strongly correlated”, “non-FL”) metals



The most prominent “bad-metal” feature: linear scaling of the resistivity



Bruin et al. Science 339, 804 (2013)

Outline

1. Coherent transport of coherent quasiparticles

1b. Drude model

Drude model

11. *Zur Elektronentheorie der Metalle;*
von P. Drude.

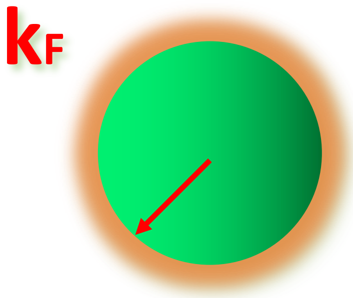
$$m^* \frac{dv}{dt} = eE - \frac{mv}{\tau}$$

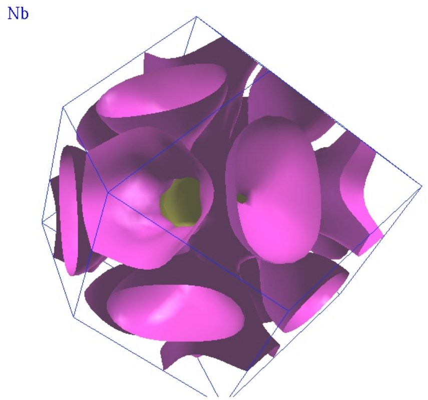
$$dc: \quad \frac{dv}{dt} = 0 \Rightarrow v = \frac{eE}{m}$$

$$j = env = \frac{e^2 n\tau}{m} E \Rightarrow$$

$$\sigma = \frac{e^2 n\tau}{m}$$

“Bloch-Drude model”





$$\hbar \frac{d\mathbf{k}}{dt} = e\mathrm{E}\hat{\mathrm{z}} - \frac{\hbar \mathbf{k}}{\tau}$$

$$dc : \frac{d\mathbf{k}}{dt} = 0 \Rightarrow k_z = \frac{eE\tau}{\hbar}$$

$$v_z = \frac{1}{\hbar} \frac{\partial \varepsilon_{\mathbf{k}}}{\partial k_z}$$

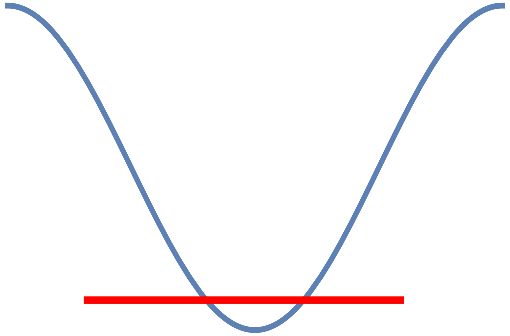
$$j_z = env_z$$

No need top solve the Boltzmann equation or use the Kubo formula!

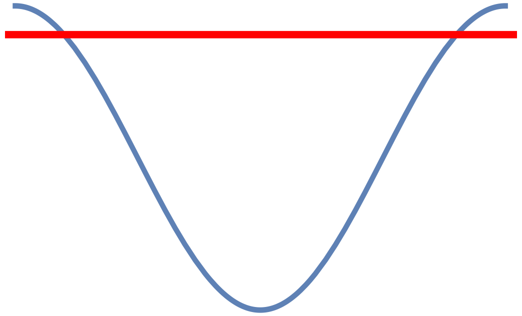
1D tight-binding model

$$\varepsilon = -t\cos(ka) \qquad v = \frac{ta}{\hbar}\sin ka = \frac{ta}{\hbar}\sin\frac{eE\tau a}{\hbar} \approx \frac{eE\tau a^2}{\hbar^2}$$

$$k = eE\tau/\hbar \qquad j = env = \frac{e^2 E\tau a^2}{\hbar^2} \Rightarrow \sigma = \frac{e^2 n\tau a}{\hbar^2}$$



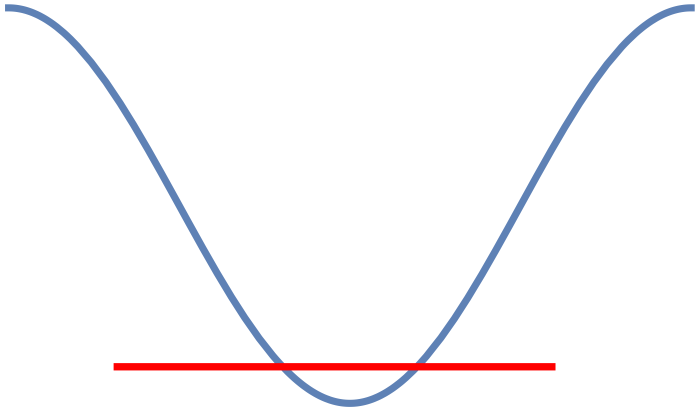
Small n→small conductivity



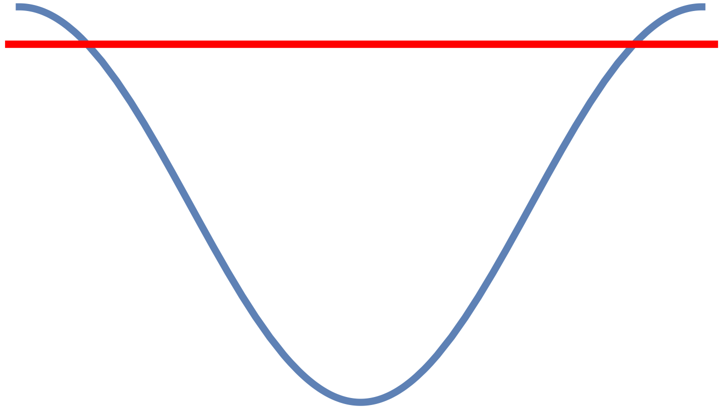
Large n→large conductivity

?

Almost empty band→small conductivity



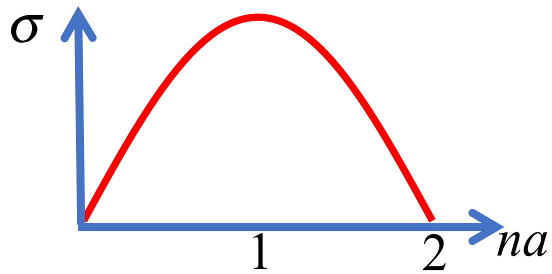
Almost full band→small conductivity



Max conductivity is at half filling!

Correct result: Boltzmann equation:

$$\sigma = \frac{2e^2}{\pi}\tau t a \sin\frac{\pi n a}{2}$$



Drude model

$$m^* \frac{dv}{dt} = eE - \frac{mv}{\tau} \qquad \rho = \frac{m^*}{ne^2\tau}$$

Matthiessen rule:

$$\frac{1}{\tau} = \frac{1}{\tau_{dis}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{ee}} + \ldots$$

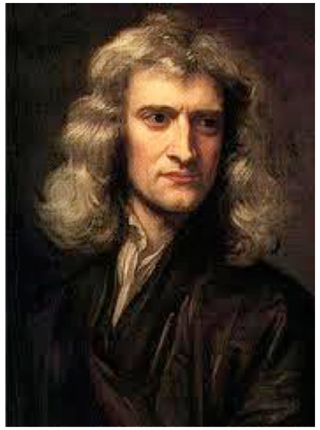
Below 10 K:

$$\frac{1}{\tau_{dis}} + \frac{1}{\tau_{ee}} + \ldots \Rightarrow \rho = \rho_0 + AT^2$$

STOP

This argument is only valid within the SCA

(SCA=Spherical Cow Approximation)



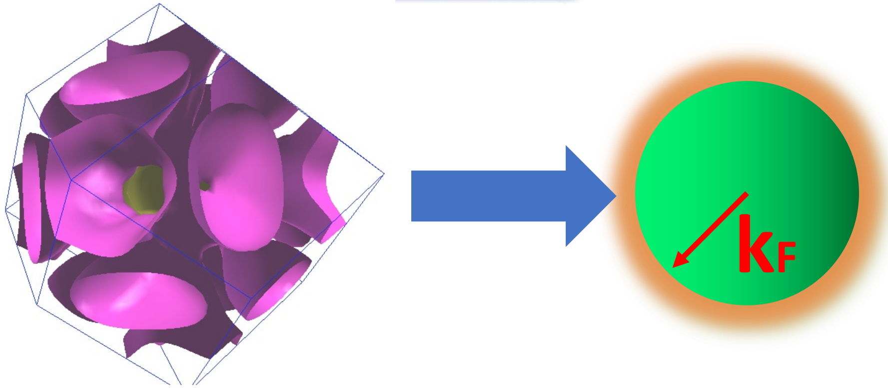
Drude model= Newton Second Law

Second law is based on Galilean invariance

(translational invariance + non-relativistic motion)

Internal forces do NOT affect the motion of the center-of-mass motion

Nb



$$\frac{dp_i}{dt} = eE + \sum_j F_{ij} \qquad F_{ij} = -F_{ji}$$

$$\frac{dP_{COM}}{dt} = eNE \Rightarrow P_{COM} = eNt$$

regardless of ee-interactions

Need to break Galilean invariance get finite resistivity

Introducing lattice explicitly will eventually give a correct result. But it is hard.

Can we get away with breaking Galilean invariance implicitly?

For example: lattice produces phonons.
Treat phonons within the Debye model:
isotropic acoustic mode

Couple Galilean-invariant electrons to a bath of isotropic acoustic phonons

$$\rho \propto \underbrace{T^5}_{1/\tau_{\mathrm{sp}}} \times \underbrace{T^2}_{\substack{1-\cos\theta \\ \approx (q/2k_F)^2}} = T^5 \text{ for } T < T_D$$

Usual framework of correlated electron systems: fermions coupled to bosons

$$L = L_F^{(0)}[c] + L_{\mathrm{B}}^{(0)}[\phi] + g\sum_q \phi_{-q}\bar{c}_{k+q\sigma}\Lambda_{\sigma\sigma'}c_{k\sigma'}$$

ϕ: real or overdamped ferro/antiferro magnon; nematic fluctuation...

Can ϕ be treated as a bath, similar to phonons?

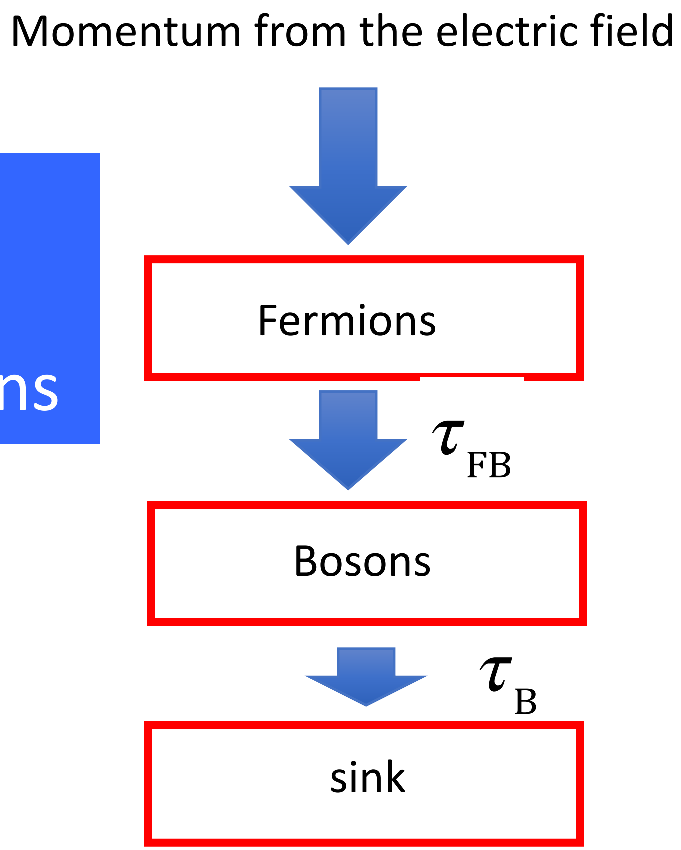
e.g. $\rho \propto T^5$ for $T < T_D$

i) Sometimes even phonons cannot be treated as a bath (phonon drag)

ii) Our boson is a (overdamped) collective mode of fermions

Bosons can be treated as a bath if $\tau_{\mathrm{B}} \ll \tau_{\mathrm{FB}}$

e.g. $\tau_{\mathrm{FB}} = \tau_{\text{e-ph}} \propto T^{-5} \gg \tau_{\mathrm{B}} = \tau_{\text{ph-disorder}} \propto T^{-(\alpha\le 4)}$ and $\rho \propto \tau_{\text{e-ph}}^{-1}$



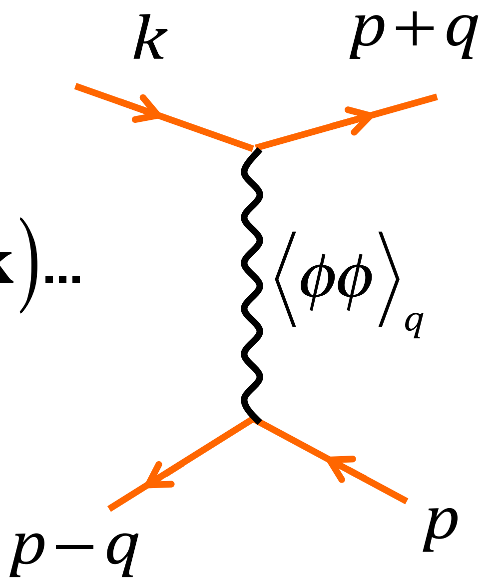
Not likely to be the case for collective modes

Back to the fermionic description

$$L = L_F^{(0)}[c] + + g \sum_{kpq} \left(\bar{c}_{k+q\sigma} \Lambda_{\sigma\sigma'} c_{k\sigma'} \right) \left(\bar{c}_{p-q\sigma''} \Lambda_{\sigma''\sigma'''} c_{k\sigma'''} \right)$$

Effective interaction mediated by boson ϕ

$$k = \left(k_0, \mathbf{k} \right) \ldots$$



Current is carried by fermions
Need to worry how it is relaxed in collisions

Three ways to get finite *dc* conductivity from e-e interactions in a non-Galilean-invariant system

Can e-e interactions alone render the conductivity finite?

1) Umklapp scattering — **Yes**

2) Interband scattering (Baber, 1937) — **No, in a generic multiband metal**
Yes, in a compensated metal

3) Single-band, anisotropic Fermi surface — **No**

"No" means that one needs another momentum-relaxing process (disorder…)

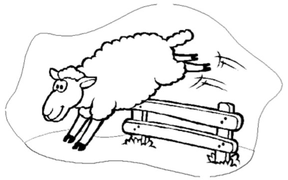
Umklapps

Rudolph Peierls

Lev Landau

Isaac Pomeranchuk

Peierls,1929: thermal conductivity of insulators



Landau & Pomeranchuk, 1936: electric conductivity of metals

$$\Psi_{\mathbf{k}}(x+a)=e^{ika}\Psi_{\mathbf{k}}(x)$$

$$k\to k+\frac{2\pi}{a}n$$

$$b=2\pi/a:\text{reciprocal lattice vector}$$

$$\mathbf{k}+\mathbf{p}=\mathbf{k}'+\mathbf{p}'+n\mathbf{b}$$

$$n=0\Rightarrow\text{Normal process}$$

$$n\neq 0\Rightarrow\text{Umklapp process}$$

$$\text{Normal processes}\Rightarrow\text{no contrbution to }\rho$$

$$\text{Umklapp processes}\Rightarrow\rho=AT^2$$

Outline

1. Coherent transport of coherent quasiparticles

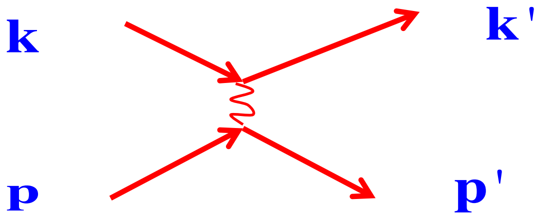
1c. Boltzmann equation→T^2 term in the resistivity

Boltzmann equation

$$f(\mathbf{k},\mathbf{r},t)$$

$$n = 2\int \frac{d^D\mathbf{k}}{(2\pi)^D} f(\mathbf{k},\mathbf{r},t),\ \mathbf{j} = e\int \frac{d^D\mathbf{k}}{(2\pi)^D}\mathbf{v}_\mathbf{k} f(\mathbf{k},\mathbf{r},t),\ \mathbf{j}_T = \int \frac{d^D\mathbf{k}}{(2\pi)^D}\varepsilon_\mathbf{k}\mathbf{v}_\mathbf{k} f(\mathbf{k},\mathbf{r},t)$$

$$\frac{\partial f_\mathbf{k}}{\partial t} + \mathbf{v}_\mathbf{k}\bullet\frac{\partial f_\mathbf{k}}{\partial \mathbf{r}} + \mathbf{F}\bullet\frac{\partial f_\mathbf{k}}{\partial \mathbf{k}} = I_c = \text{collision integral}$$



$$\mathbf{F} = e\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B}$$

$$\text{Disorder: } \left(I_c\right)_{\text{imp}} = -\frac{f_\mathbf{k} - \langle f_\mathbf{k}\rangle}{\tau} \Rightarrow \frac{f_\mathbf{k} - f_{0\mathbf{k}}}{\tau}$$

OK for the conductivity
but not for other purposes

$$\text{ee Interaction: } \left(I_c\right)_{\text{ee}} = \sum_{\mathbf{p}',\mathbf{k}',\mathbf{p}'\mathbf{b}} W_{\mathbf{kp}\to\mathbf{k}'\mathbf{p}'}\left[f_{\mathbf{k}'}f_{\mathbf{p}'}\left(1-f_\mathbf{k}\right)\left(1-f_\mathbf{p}\right) - f_\mathbf{k}f_\mathbf{p}\left(1-f_{\mathbf{k}'}\right)\left(1-f_{\mathbf{p}'}\right)\right]$$

$$\times\,\delta_{\mathbf{k}+\mathbf{p},\mathbf{k}'+\mathbf{p}'+\mathbf{b}}\,\delta\left(\varepsilon_\mathbf{k}+\varepsilon_\mathbf{p}-\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{p}'}\right)$$

Weak external fields: linearized Boltzmann equation

No magnetic field

$$f_{\mathbf{k}} = f_{0\mathbf{k}} + T\left(-\frac{\partial f_{0\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}}\right) g_{\mathbf{k}}$$

$$\mathbf{F}\bullet\frac{\partial f_{\mathbf{k}}}{\partial \mathbf{k}} \approx e\mathbf{E}\frac{\partial f_{0\mathbf{k}}}{\partial \mathbf{k}} = e\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}}\frac{\partial f_{0\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}}$$

$$\frac{\partial f_{\mathbf{k}}}{\partial t} + \mathbf{v}_{\mathbf{k}}\bullet\frac{\partial f_{\mathbf{k}}}{\partial \mathbf{r}} + e\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}}\frac{\partial f_{0\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}} = \left(I_c\right)_{\text{imp}} + \left(I_c\right)_{\text{ee}}$$

$$\left(I_c\right)_{\text{ee}} = -\sum_{\mathbf{p}',\mathbf{k}',\mathbf{p}'\mathbf{b}} W_{\mathbf{kp}\to\mathbf{k}'\mathbf{p}'} f_{0\mathbf{k}'} f_{0\mathbf{p}'}\left(1-f_{0\mathbf{k}}\right)\left(1-f_{0\mathbf{p}}\right)\left[g_{\mathbf{k}} + g_{\mathbf{p}} - g_{\mathbf{k}'} - g_{\mathbf{p}'}\right]\times\delta_{\mathbf{k}+\mathbf{p},\mathbf{k}'+\mathbf{p}'+\mathbf{b}}\,\delta\left(\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{p}'}\right)$$

Arbitrary dispersion: $\mathbf{v}_{\mathbf{k}} = \nabla_{\mathbf{k}}\varepsilon_{\mathbf{k}} \neq \mathbf{k}/m$

BUT no Umklapps

Statement: in a single-band system, e-e interaction alone cannot render *dc* conductivity finite

1) Proof via the Boltzmann equation

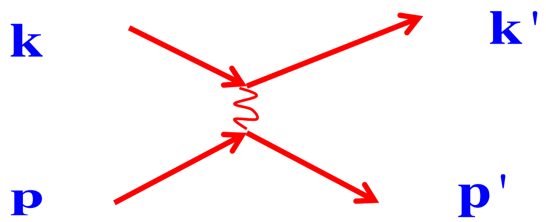
$$e\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}}\frac{\partial f_{0\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}} = -\sum_{\mathbf{p}',\mathbf{k}',\mathbf{p}'b} W_{\mathbf{kp}\to\mathbf{k}'\mathbf{p}'} f_{0\mathbf{k}'} f_{0\mathbf{p}'}\left(1-f_{0\mathbf{k}}\right)\left(1-f_{0\mathbf{p}}\right)\left[g_{\mathbf{k}}+g_{\mathbf{p}}-g_{\mathbf{k}'}-g_{\mathbf{p}'}\right]\times\delta_{\mathbf{k}+\mathbf{p},\mathbf{k}'+\mathbf{p}'}\delta\left(\varepsilon_{\mathbf{k}}+\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{p}'}\right)$$

$$\mathbf{C}\bullet\left[\mathbf{k}+\mathbf{p}-\mathbf{k}'-\mathbf{p}'\right]=0$$

Suppose we found a solution: $g^{(1)}_{\mathbf{k}}$.

But then $g^{(2)}_{\mathbf{k}} = g^{(1)}_{\mathbf{k}} + \mathbf{C}\bullet\mathbf{k}$ with $\forall$ $\mathbf{C}$ is also a solution.

$|\mathbf{C}|$ can be infinitely large $\Rightarrow \sigma$ can arbitrarily large



$$\mathbf{k}+\mathbf{p}=\mathbf{k}'+\mathbf{p}'$$

Momentum conservation

$$\partial_t P + \partial_{x_j}\Pi_{ij} = -eNE_i$$

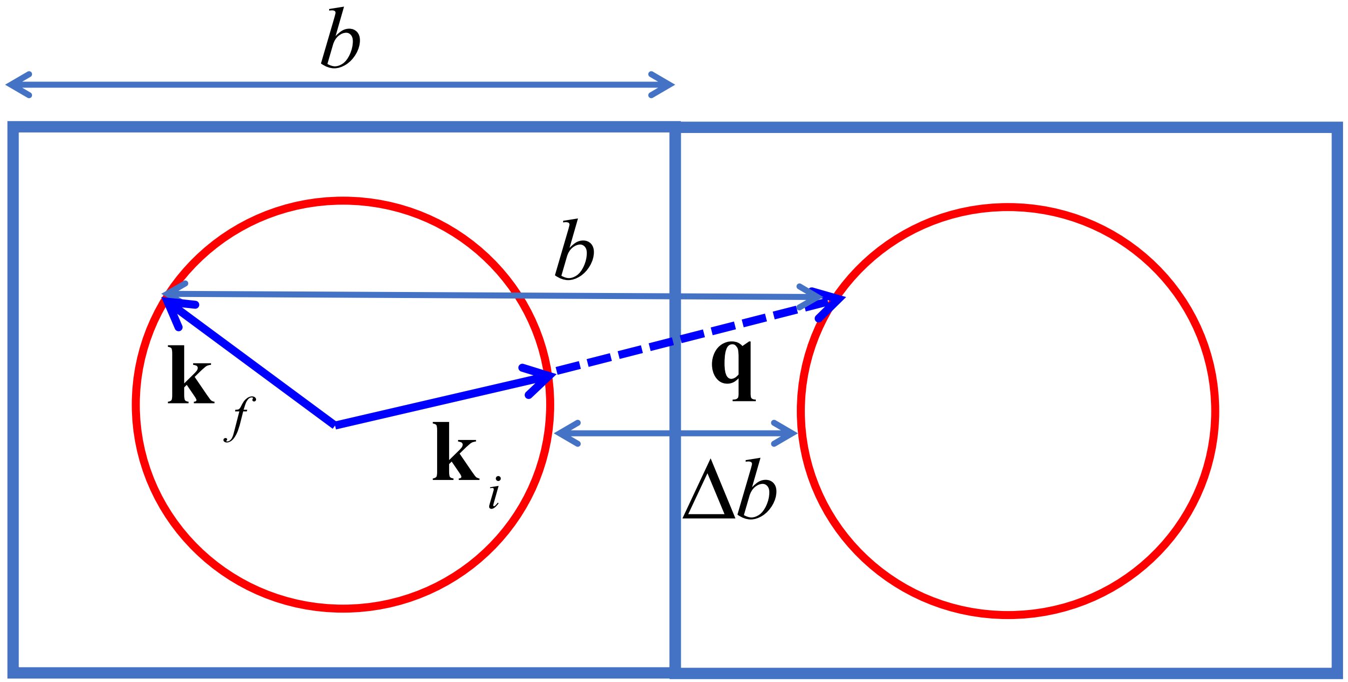
$$\Pi_{ij} = \int_{\mathbf{k}} v_i k_j f_{\mathbf{k}}$$

2) Diagrams for the Kubo formula (Maebashi & Fukuyama, JPSJ 1997)

$$\sigma'\left(\omega\right) = D\delta\left(\omega\right)$$

3) Memory matrix formalism : zero mode of the memory matrix

Example: umklapp scattering by an acoustic phonon

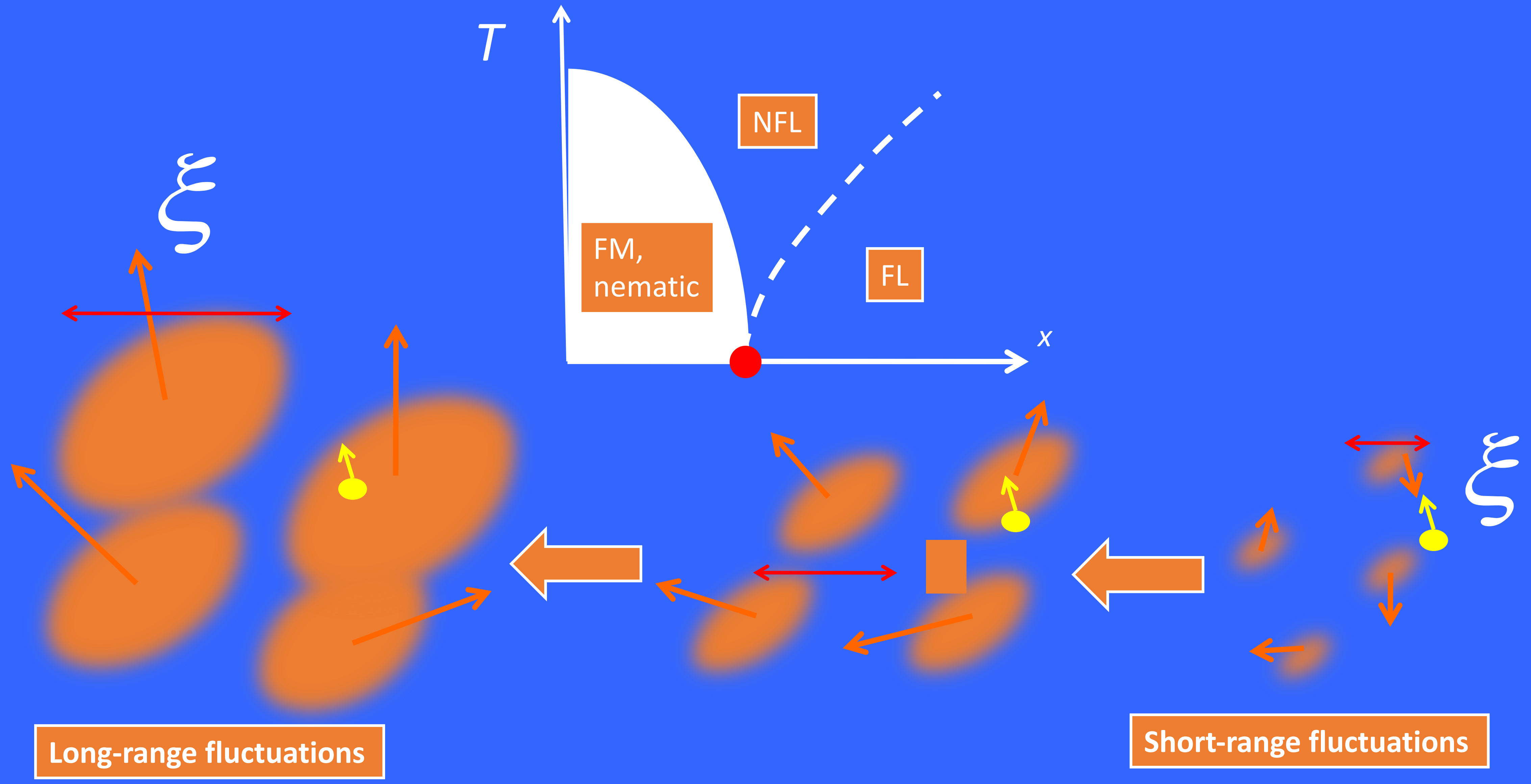


$$\mathbf{k}_f = \mathbf{k}_i + \mathbf{q} - 2\pi\hat{x}$$

$$\mathbf{j} = \sum_{\mathbf{k}\in \text{1st BZ}} e\mathbf{v}_{\mathbf{k}} \delta f_{\mathbf{k}}$$

Freeze-out at low T: $q \sim T/s \ll \Delta b$

$$\rho \propto \exp\left(-s\Delta b / T\right)$$



Effective interaction: exchange by diverging order parameter fluctuations→

FERMI-LIQUID BREAKDOWN

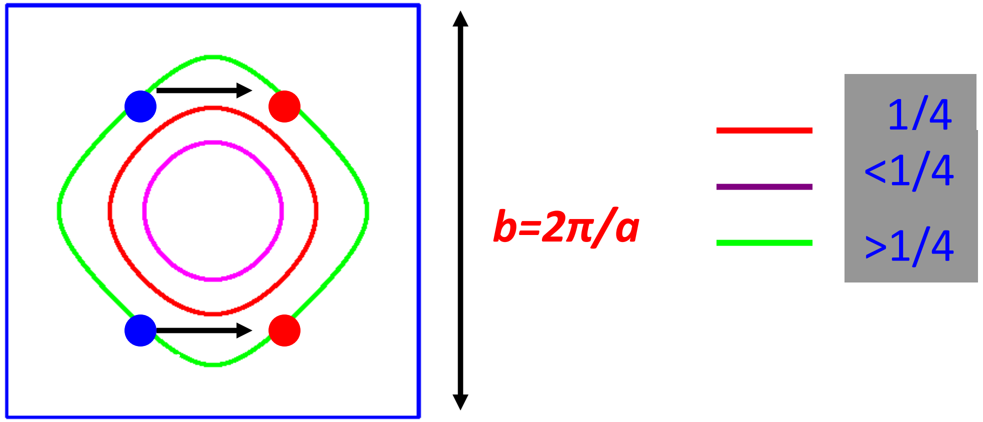
Two conditions for Umklapps from electron-electron interaction

a) large Fermi surface: $k_F \sim b$

b) short-range interaction (well-screened Coulomb): $q \sim \kappa \sim k_F \sim b$

$$\mathbf{k}+\mathbf{p}-\mathbf{k}'-\mathbf{p}'=\mathbf{b}$$

$$\max|\mathbf{k}+\mathbf{p}-\mathbf{k}'-\mathbf{p}'|=4k_F \geq b$$



Both conditions are satisfied in typical metals➔

$$\rho = A_U T^2$$

Condition a) is violated in low-density Fermi liquids
(degenerate semiconductors, semimetals)
Condition b) is violated in systems with long-range interaction, e.g.,
near Pomeranchuk instabilities

Typical momentum transfer $\bar{q} \ll k_F \sim b$

τ_{sp} : single-particle lifetime

τ_{tr} : momentum relaxation time

$$\frac{1}{\tau_{\mathrm{tr}}} = \frac{1}{\tau_{\mathrm{sp}}}\left(\frac{\bar{q}}{k_F}\right)^2 \ll \frac{1}{\tau_{\mathrm{sp}}}$$

$\mathbf{k}+\mathbf{p}=\mathbf{k}'+\mathbf{p}'+\mathbf{b}$

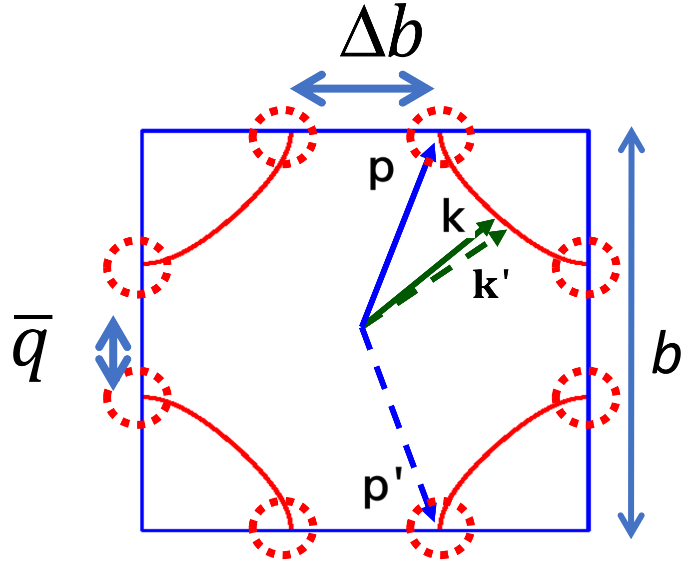
small-angle scattering: $\mathbf{k} \approx \mathbf{k}'$

$\Rightarrow \mathbf{p}\text{-}\mathbf{p}' \approx \mathbf{b}$

Possible only at "Umklapp hot spots"

Yudson, Chubukov, DM PRL 2011
Pal, Yudson, DM Lith. J. Phys. 2012
Erratum: D->D-1
X. Wang and Berg 2019

Generic FS: $\Delta b \sim b$



$$\frac{1}{\tau_u} = \frac{1}{\tau_{\mathrm{tr}}} \times \left(\frac{\bar{q}}{b}\right)^{D-1}$$

$z=3$ criticality: time $\propto (\text{space})^z$

time$=1/T$, space$=1/q$

$\bar{q} \propto T^{1/z} = T^{1/3}$

$\tau_{\mathrm{sp}}^{-1} \propto T^{\frac{D}{3}}$; $\tau_{\mathrm{tr}}^{-1} \propto T^{\frac{D+2}{3}}$

$$\tau_{\mathrm{U}}^{-1} \propto T^{\frac{2D+1}{3}} \propto \begin{cases} T^{7/3}, & 3\mathrm{D} \\ T^{5/3}, & 2\mathrm{D} \end{cases}$$

Three ways to get a finite contribution to the *dc* conductivity from e-e interactions in a non-Galilean-invariant system

1) Umklapp scattering

2) Baber (interband) scattering

3) Anisotropic Fermi surface

Multi-band metal

Baber 1937

$$m_1\frac{dv}{dt}=e_1E-\underbrace{\frac{m_1v_1}{\tau_1}}_{\text{intra-band}}-\underbrace{\overbrace{\gamma}^{\propto T^2}\, n_2\left(v_1-v_2\right)}_{\text{interband}}$$

$$m_2\frac{dv}{dt}=e_2E-\frac{m_2v_2}{\tau_2}-\gamma n_1\left(v_1-v_2\right)$$

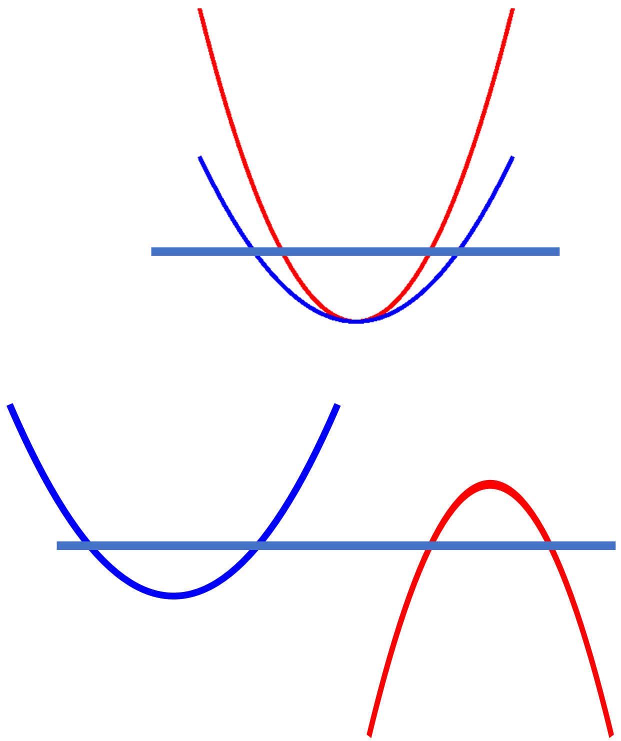
1) $e_1e_2=e^2>0$ electron or hole bands

Still requires intra-band relaxation

2) $e_1e_2<0$ AND $n_1=n_2$: compensated metal

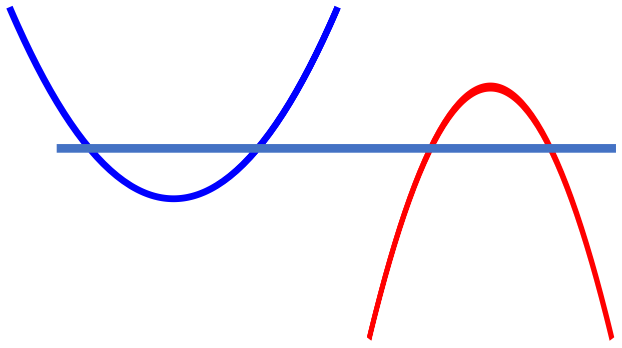
Does not require intra-band relaxation

Two parabolic bands: each of the bands is Galilean-invariant→Drude model works



Compensated metals: wide class of materials

i) Metals with even # *e^-/u.cell* (Zn, Mg, Cd, Pd…)
ii) Semimetals: Bi, Sb, graphite,Weyl II semimetals (WP_2)
iii) Parent states of Fe-based superconductors (remain compensated on isovalent doping)
iv) Weak ferromagnets ($Pd_{1-x}Ni_x$, $ZrZn_2$) exhibiting quantum-critical phenomena



Quick calculation: a compensated metal without intra-band relaxaton

$$0 = eE - \gamma n\left(v_1 - v_2\right)$$

$$0 = -eE + \gamma n\left(v_1 - v_2\right)$$

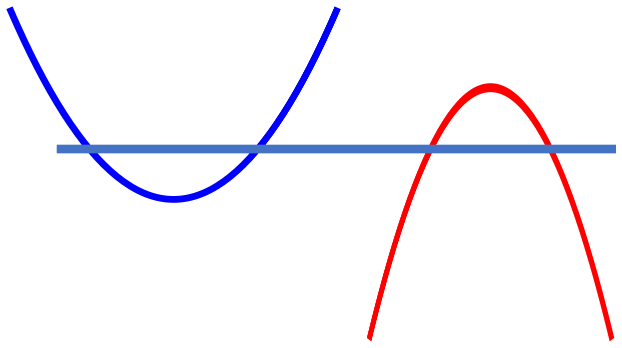
Only one equation instead of two!

$$v_1 - v_2 = eE / \gamma n$$

v_1 and v_2 cannot be found separately but they are not needed:

$$j = en\left(v_1 - v_2\right) = e^2 E / \gamma$$

$$\Rightarrow \sigma = e^2 / \gamma \propto T^{-2} \text{ for a FL}$$



dc transport in two-band metal

band 1: light; band 2: heavy

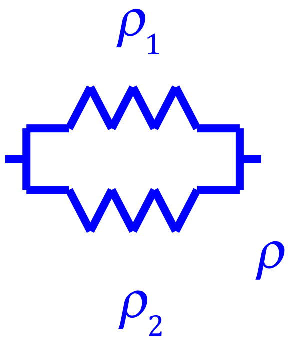
$$\rho_1(T=0) \ll \rho_2(T=0)$$

$$0 = e_1 E - \underbrace{\frac{m_1 v_1}{\tau_1}}_{\text{impurities}} - \underbrace{\gamma}_{\propto T^\alpha} n_2 (v_1 - v_2)$$

FL: $\alpha = 2$

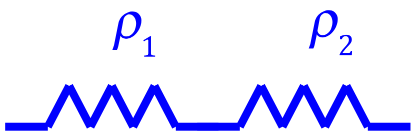
$$0 = e_2 E - \underbrace{\frac{m_1 v_1}{\tau_2}}_{\text{impurities}} + \gamma n_1 (v_1 - v_2)$$

$T \to 0:$

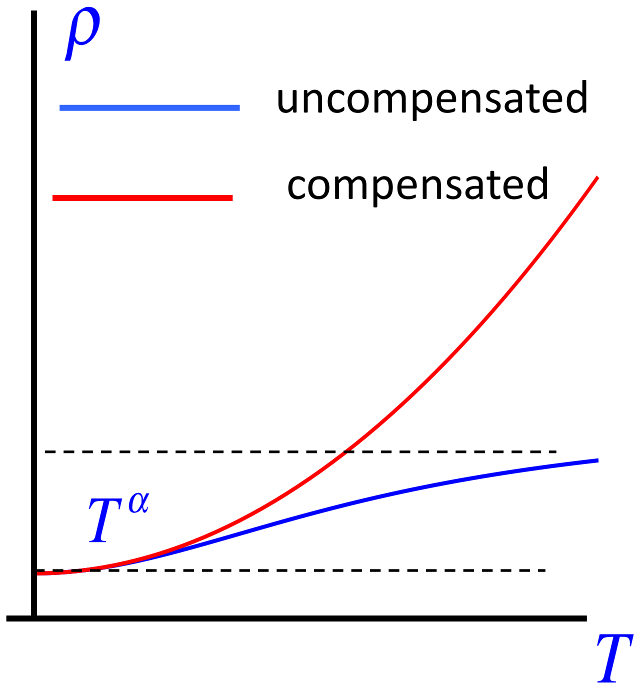


$$\rho(T=0) = \frac{1}{\sigma_1 + \sigma_2} \approx \rho_1$$

$T \to \infty:$



$$\rho(T=\infty) = \rho_1 + \rho_2 \approx \rho_2$$



$$\rho|_{T=0} = 4\pi \frac{1}{\Omega_1^2 \tau_1 + \Omega_2^2 \tau_2},$$

$$\Omega_{1,2}^2 = 4\pi e^2 n_{1,2}/m_{1,2}$$

$$\rho|_{T\to\infty} = \frac{4\pi}{\Omega_0^2}\left(\frac{1}{\tau_1}\frac{1}{1+\frac{n_2 m_2}{n_1 m_1}} + \frac{1}{\tau_2}\frac{1}{1+\frac{n_1 m_1}{n_2 m_2}}\right).$$

$$\Omega_0^2 = 4\pi \frac{(e_1 n_1 + e_2 n_2)^2}{n_1 m_1 + n_2 m_2}$$

Compensated metal: $e_1 n_1 + e_2 n_2 = 0$

$$\Rightarrow \Omega_0 = 0 \Rightarrow \rho(T=\infty) = \infty$$

Scattering mechanism in a compensated metal is similar to that in undoped graphene

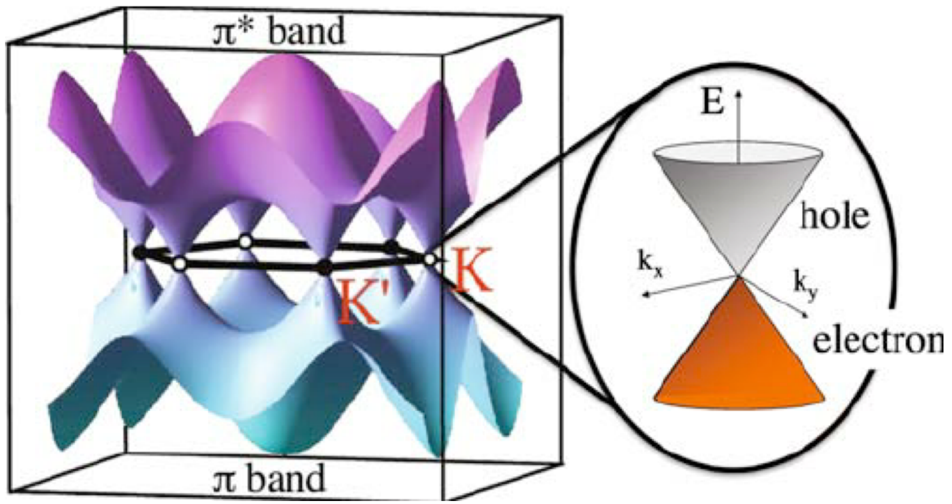
$$H = v_0 \vec{\sigma} \cdot \mathbf{k}$$

distribution function is a 2×2 matrix

$$\partial_t \hat{f} + i[H, \hat{f}] + e\mathbf{E} \cdot \nabla_{\mathbf{p}} \hat{f} = I[\hat{f}]$$

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h}\left\{\dfrac{\pi}{2} + \mathcal{O}\left[\dfrac{1}{\ln(\Lambda/\hbar\omega)}\right]\right\}, & \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left\{0.760 + \mathcal{O}\left[\dfrac{1}{|\ln(\alpha(T))|}\right]\right\}, & \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

Fritz, Schmalian, Mueller, Sachdev PRB 2008



Can be understood as a compensated (semi) metal with $n_e = n_h \propto T$ and $\gamma \propto T$

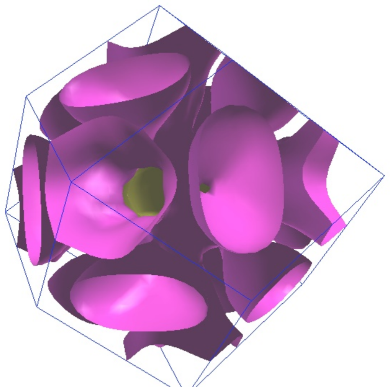
Three ways to get a finite contribution to the *dc* conductivity from e-e interactions in a non-Galilean-invariant system

1) Umklapp scattering

2) Baber (interband) scattering

3) Anisotropic Fermi surface
(ee+impurities or finite frequency)

e-e interaction + impurities



Anisotropic FS

Without umklapps or disorder the conductivity is infinite
Add disorder but keep umklapps out

$$e\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}}\frac{\partial f_{0\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}}=\left(I_c\right)_{\mathrm{imp}}+\left(I_c\right)_{\mathrm{ee}}=-\frac{f_{\mathbf{k}}-f_{0\mathbf{k}}}{\tau}+\left(I_c\right)_{\mathrm{ee}}$$

ee collisions conserve the momentum→ $\int_{\mathbf{k}}\left(I_c\right)_{\mathrm{ee}}\mathbf{k}=0$

Suppose the system is Galilean-invariant
(remain so on averaging over disorder)

$$\varepsilon_{\mathbf{k}}=\frac{k^2}{2m}\Rightarrow\mathbf{v}_{\mathbf{k}}=\frac{\mathbf{k}}{m}$$

$$\Rightarrow\int_{\mathbf{k}}\left(I_c\right)_{\mathrm{ee}}\mathbf{v}_{\mathbf{k}}=0$$

Multiply the B.E. by $e\mathbf{v}_{\mathbf{k}}$ and integrate over $\mathbf{k}$

$$e^2\int_{\mathbf{k}}\mathbf{v}_{\mathbf{k}}(\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}})\frac{\partial f_{0\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}}=-\frac{\mathbf{j}}{\tau}\Rightarrow\sigma=\frac{ne^2\tau}{m}$$

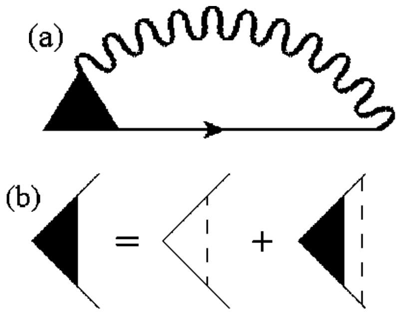
ee interaction in a dirty but Galilean-invariant system does not affect the conductivity

“ee interaction in a dirty but Galilean-invariant system does not affect the conductivity”

Valid only in the Boltzmann equation framework where different sources of scattering act independently

Effects beyond the Boltzmann equation:

1) Quantum corrections
   (weak localization, Altshuler-Aronov/Zala-Narozhny-Aleiner, Aslamazov-Larkin…)
2) Hydrodynamic (viscous) correction in bulk samples (Hruska and Spivak PRB 2002; Andreev,Kivelson, Spivak 2011)
3) Disorder in the bosonic mass (Patel & Sachdev 2014)



$$\rho = \rho_{\mathrm{imp}} + \delta\rho(T)$$

$$\text{with } \delta\rho(T) \propto \text{disorder}$$

Violation of the Matthiesen rule: resistors are not connected in series

Altshuler-Aronov/Zala-Narozhny-Aleiner

Wiedemann-Franz Law (1853)

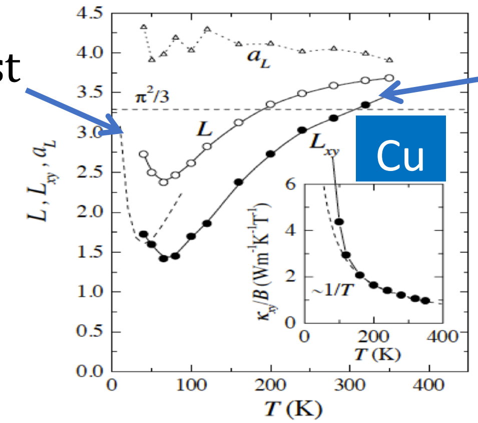
$$\frac{\kappa}{T\sigma} = L_0 = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

"Hallmark of the FL behavior"

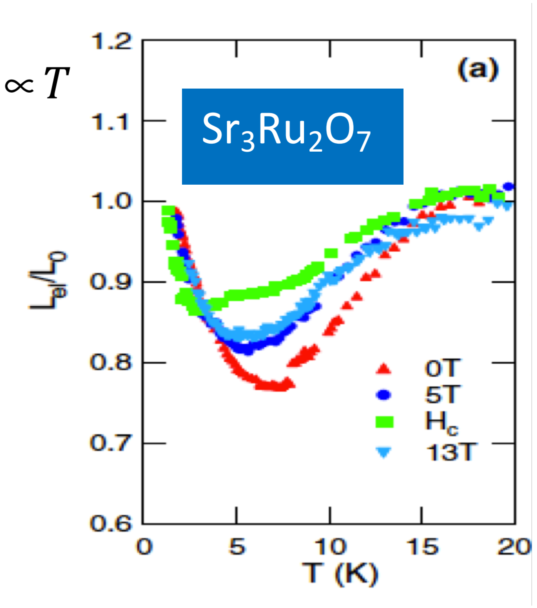
Not quite: hallmark of *elastic* scattering

energy relaxation rate <<
momentum relaxation rate

$\rho = \text{const}$



$\rho \propto T$



Zhang et al. PRL 2000

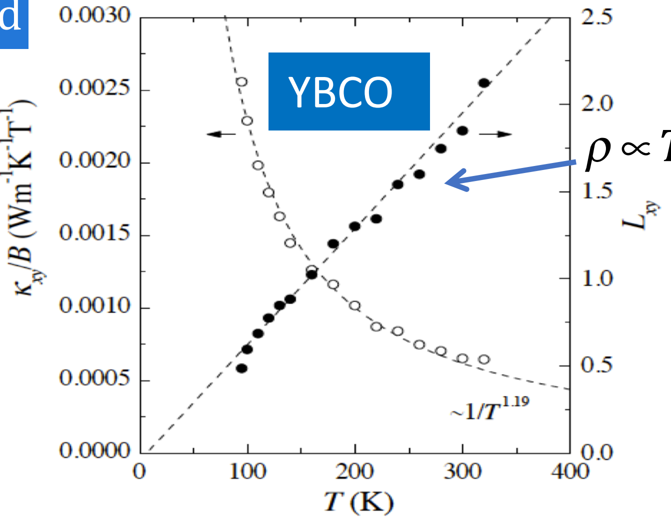
Ronning et al. PRL 2006

Classical boson: $N \approx T / T_D$

$T > "T_D":\ \delta\varepsilon \le T_D \Rightarrow$ diffusion in energy

every collision relaxes momentum

scattering from a thermally disordered field



$\rho \propto T$

WF is violated
Inconsistent with scattering from classical bosons:
downward violation:
energy is relaxed faster than the momentum

Can e-e interactions alone render the resistivity finite and control its *T*-dependence?

1) Umklapp scattering

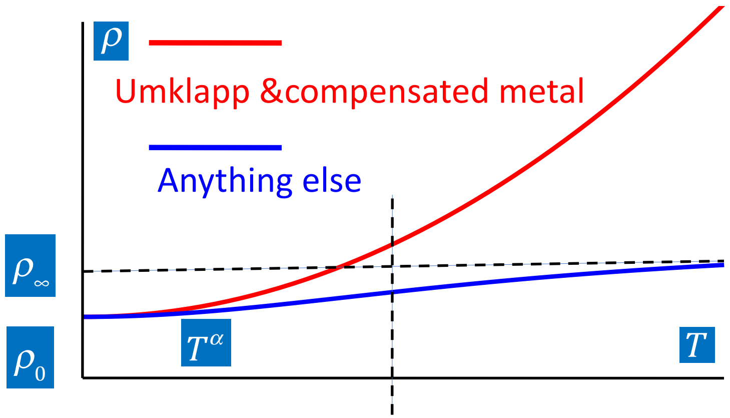
Yes, but it is suppressed near Pomeranchuk quantum criticality and short-circuited by cold regions of the FS

1) Interband scattering

Generic multiband metal: need impurities the fix the low- and high-T values of resistivity
Compensated metal: no saturation at higher T

1) Single-band, anisotropic Fermi surface

~generic multiband



Outline

1. Coherent transport of coherent quasiparticles

1a. Good vs bad (metals)

1b. Drude model and its pitfalls

1c. Resistivity from e-e interaction:

umklapp scattering

normal scattering in i) multiband, ii) compensated, iii)anisotropic metals

1d Optical conductivity

1e The puzzle of charge transport in STO

2. Coherent transport of incoherent quasiparticles

2a. Charge and thermal transport near a ferromagnetic quantum critical point

2b. Which mass enters the conductivity?

Non-Galilean-invariant system: ee interaction affects the conductivity even within the Boltzmann equation

R. Gurzhi
Sov. Phys. Uspekhi 1968

$$\mathbf{v}_{\mathbf{k}} \neq \frac{\mathbf{k}}{m} \Rightarrow \int_{\mathbf{k}} \left(I_c\right)_{\mathrm{ee}} \mathbf{k} = 0 \text{ but } \int_{\mathbf{k}} \left(I_c\right)_{\mathrm{ee}} \mathbf{v}_{\mathbf{k}} \neq 0$$

Momentum is conserved but current is not

$$e^2 \int_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} (\mathbf{E} \bullet \mathbf{v}_{\mathbf{k}}) \frac{\partial f_{0\mathbf{k}}}{\partial \varepsilon_{\mathbf{k}}} = -\frac{\mathbf{j}}{\tau} + \underbrace{e \int_{\mathbf{k}} \left(I_c\right)_{\mathrm{ee}} \mathbf{v}_{\mathbf{k}}}_{\text{current relaxation by ee interaction}}$$

Effect of ee interaction on the conductivity

1) Low temperature: frequent ei collisions, rare ee collisions→ perturbation theory in ee scattering

$$e\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}}\frac{\partial f_{0\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}}=-\frac{f_{\mathbf{k}}-f_{0\mathbf{k}}}{\tau}+\left(I_c\right)_{\text{ee}}[f_{\mathbf{k}}]$$

Drop $\left(I_c\right)_{\text{ee}}$ and solve

$$e\mathbf{E}\bullet\mathbf{v}_{\mathbf{k}}\frac{\partial f_{0\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}}=-\frac{f^{\text{imp}}-f_{0\mathbf{k}}}{\tau}$$

Substitute $f_{\mathbf{k}}=f^{\text{imp}}+\delta f^{\text{ee}}$ into the RHS $\Rightarrow$

$$\delta f^{\text{ee}}=\tau\left(I_c\right)_{\text{ee}}[f^{\text{imp}}]\sim\frac{\tau}{\tau_{\text{ee}}}f^{\text{imp}}\ll f^{\text{imp}}$$

Find the correcton to the current and to the conductivity

Correction to the residual resistivity due to ee

$$\rho = \rho_{\text{imp}} + \rho_{\text{ee}}$$

FL: $\rho_{\text{ee}} = T^2 \int_{\mathbf{k}\in \text{FS}} dA_{\mathbf{k}} \int_{\mathbf{p}\in \text{FS}} dA_{\mathbf{p}} \int \frac{d^D q}{(2\pi)^D} W_{\mathbf{kp}\to\mathbf{k+qp-q}} \left(\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{p}} - \mathbf{v}_{\mathbf{k-q}} - \mathbf{v}_{\mathbf{p+q}}\right)^2 \delta\left(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k-q}}\right) \delta\left(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p+q}}\right)$

Check: Galilean-invariant system $\mathbf{v}_{\mathbf{k}} = \frac{\mathbf{k}}{m} \ldots \Rightarrow \left(\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{p}} - \mathbf{v}_{\mathbf{k-q}} - \mathbf{v}_{\mathbf{p+q}}\right)^2 = 0 \Rightarrow \rho_{\text{ee}} = 0 \ \vee$

For a generic FS, $\rho = \rho_{\text{imp}} + AT^2$ but $AT^2 \ll \rho_{\text{imp}}$

NB: A is independent of disorder

``a la Matthiesen"

Only at low T!

2) High temperatures: frequent ee collisions, rare ei collisions but…

ee collisions CANNOT render the conductivity finite on their own

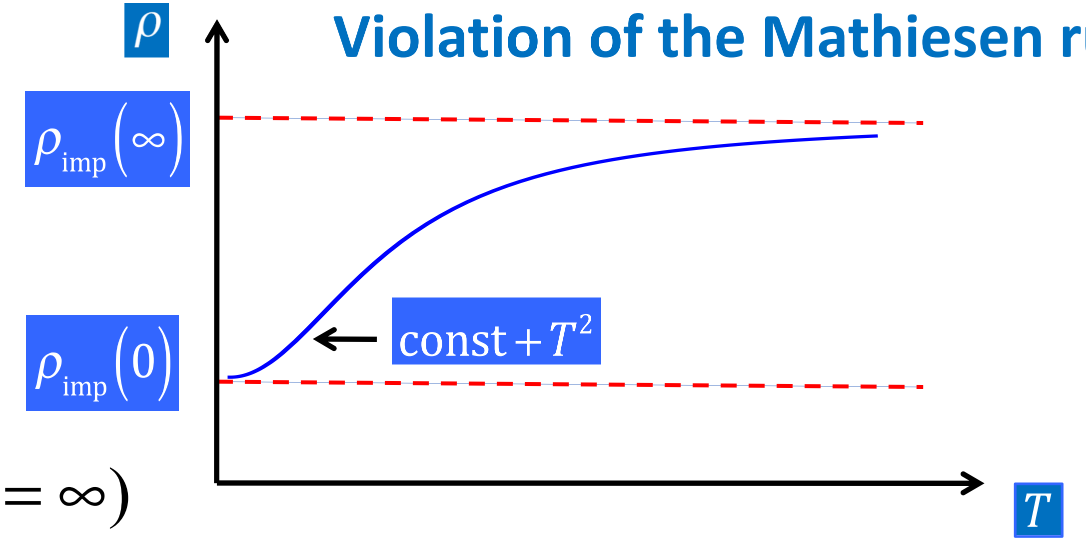
ee collisions equilibrate electron liquid whose center of mass flows with the velocity determined by disorder

$$f = f_0\left(\varepsilon_{\mathbf{k}} - \mathbf{u}\cdot\mathbf{k}\right)$$

$$\sigma_{ij}\left(T=0\right) = e^2 v_F \tau_{\text{imp}} \left\langle v_i v_j \right\rangle_{\text{FS}}$$

$$\sigma_{ij}\left(T\to\infty\right) = e^2 v_F \tau_{\text{imp}} \sum_l \frac{\left\langle v_i k_l \right\rangle \left\langle v_j k_l \right\rangle}{\left\langle k_l^2 \right\rangle}$$

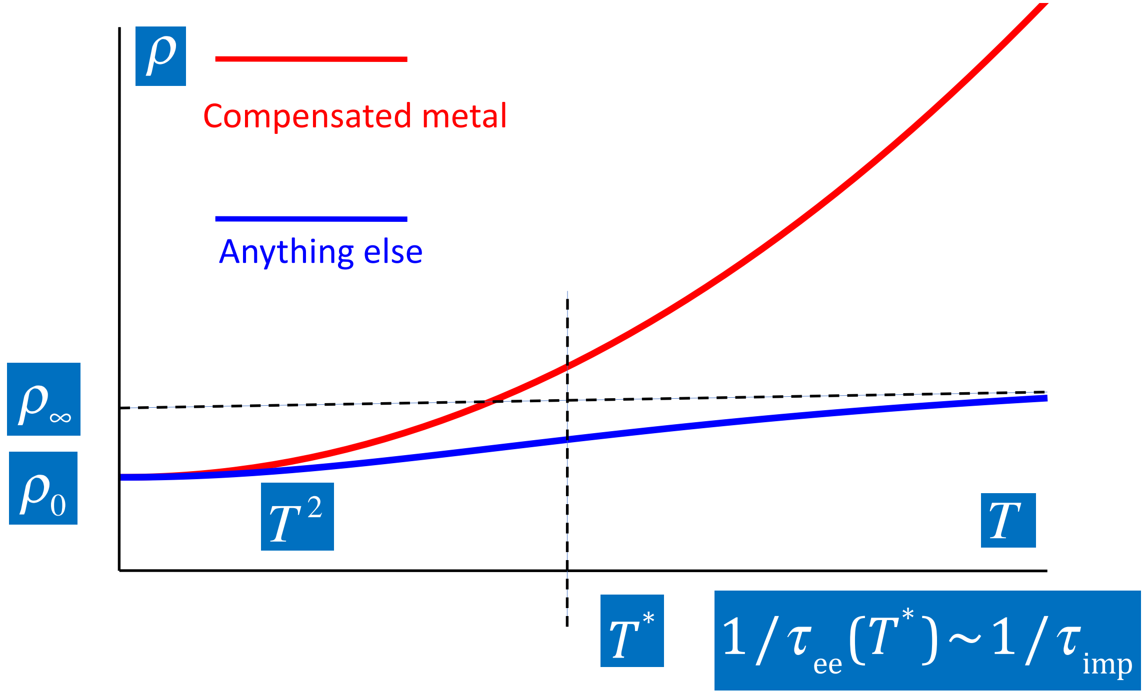
Violation of the Mathiesen rule



Sphere (or ellipsoid): $\sigma_{ij}\left(T=0\right) = \sigma_{ij}\left(T=\infty\right)$

Otherwise $\sigma_{ij}\left(T=0\right) < \sigma_{ij}\left(T=\infty\right)$ **but generically** $\sigma_{ij}\left(T=0\right) \sim \sigma_{ij}\left(T=\infty\right)$

Summary: non-Galilean-invariant system: normal ee collisions +disorder



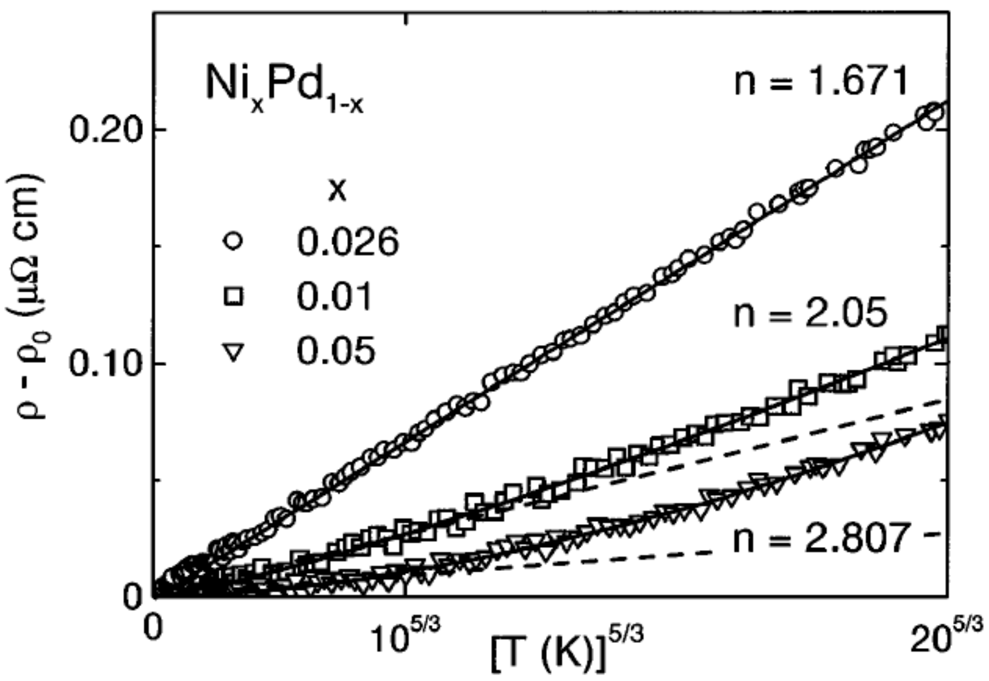
Both ρ_∞ and ρ_0 are determined by disorder (in reality, phonons mask saturation at high T)

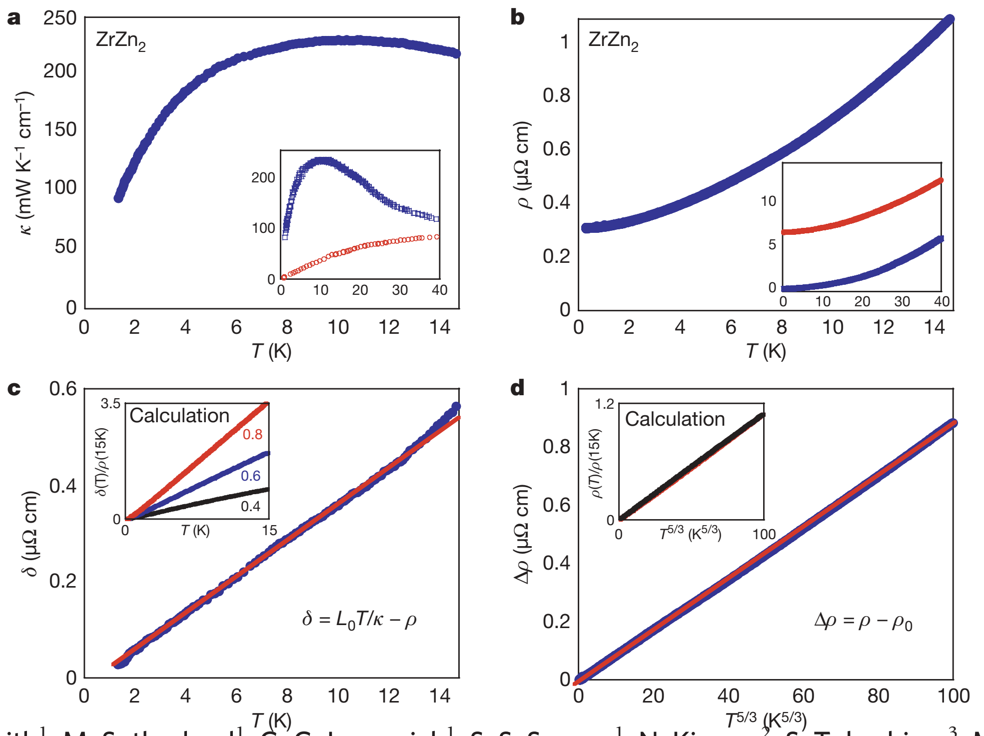
Difference between the multiband- and single-band case:

1) multiband: $\rho_\infty/\rho_0 \propto m_{\text{heavy}}/m_{\text{light}} \gg 1$

$\Rightarrow$ true scaling regime $\rho_0 \ll \rho_{\text{ee}}(T) \ll \rho_\infty$

2) single-band: $\rho_\infty \sim \rho_0$; $\Rightarrow$ no true scaling regime $\rho_0 \ll \rho_{\text{ee}}(T) \ll \rho_\infty$





R. P. Smith[1], M. Sutherland[1], G. G. Lonzarich[1], S. S. Saxena[1], N. Kimura[2], S. Takashima[3], M. Nohara[3] & H. Takagi[3,4]

M. Nicklas, M. Brando, G. Knebel, F. Mayr, W. Trinkl, and A. Loidl

PRL **82,** 4268 (1999)

$$\rho = \rho_0 + \rho(T)$$

Nature 455, 1220 (2008)

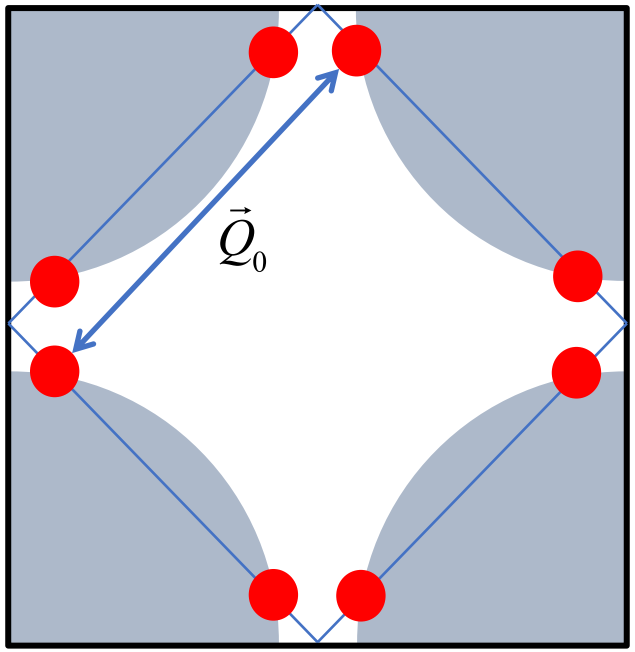
$\rho(T)/\rho_0 < 15\%$ (D. Belitz, private comm.)

$\rho(T)/\rho_0 \approx 3$

Can be attributed to single-band mechanism

Need other mechanisms

Another role of impurities:
Getting out of the Hlubina-Rice conundrum



$1/\tau_{\text{hot}} \gg 1/\tau_{\text{cold}}$

SDW (z=2) criticality in 2D: $1/\tau_{\text{hot}} \propto \sqrt{T}$

Linear size of the hot spot: $\bar{q} \propto \sqrt{T}$

Naively, $\rho \propto (1/\tau_{\text{hot}})\bar{q} \propto T$

Not so fast (Hlubina & Rice PRB 1995): $\sigma = e^2 \left\langle v_F^2 \tau(\mathbf{k}) \right\rangle_{\text{FS}}$

Cold regions short-circuit hot regions: $\sigma \propto \tau_{\text{cold}} \propto T^{-2}$

Rosch (PRL 1999): scattering time with impurities

$$\tau(\mathbf{k}) = \frac{1}{1/\tau_{\text{imp}} + 1/\tau_{\text{hot}}(\mathbf{k})} \approx \tau_{\text{imp}} - \frac{\tau_{\text{imp}}^2}{\tau_{\text{hot}}(\mathbf{k})} + \ldots \text{ for } \tau_{\text{imp}} \ll \tau_{\text{hot}}(\mathbf{k})$$

$$\rho = \rho_{\text{imp}} + \frac{1}{e^2 \left\langle v_F^2 \right\rangle} \left\langle \frac{v_F^2}{\tau_{\text{hot}}(\mathbf{k})} \right\rangle = \rho_{\text{imp}} + CT$$

at least for $CT \ll \rho_{\text{imp}}$

Optical conductivity

Finite frequency ~ impurity scattering

$$1/\tau_{\text{imp}} \Rightarrow \omega$$

Finite *dc* conductivity: need umklapps or electron-hole scattering in a compensated metal
optical conductivity: a single but anisotropic Fermi surface suffices

$$-i\omega\delta f + e\mathbf{v}_{\mathbf{k}}\cdot\mathbf{E}\frac{\partial f_0}{\partial\varepsilon_{\mathbf{k}}} = \left(I_c\right)_{ee}\left[\delta f\right]$$

Iterate w.r.t. $\left(I_c\right)_{ee}$

$$-i\omega\delta f^{(0)} + e\mathbf{v}_{\mathbf{k}}\cdot\mathbf{E}\frac{\partial f_{0\mathbf{k}}}{\partial\varepsilon_{\mathbf{k}}} = 0$$

$$-i\omega\delta f^{(1)} = \left(I_c\right)_{ee}\left[\delta f^{(0)}\right]$$

$$\left(I_c\right)_{ee}[\delta f] = \sum_{\mathbf{k'pp'}} \bar{W}_{\mathbf{kp}\to\mathbf{k'p'}}\left(\delta f^{(0)}{}_{\mathbf{k}} + \delta f^{(0)}{}_{\mathbf{p}} - \delta f^{(0)}{}_{\mathbf{k'}} - \delta f^{(0)}{}_{\mathbf{p'}}\right)$$

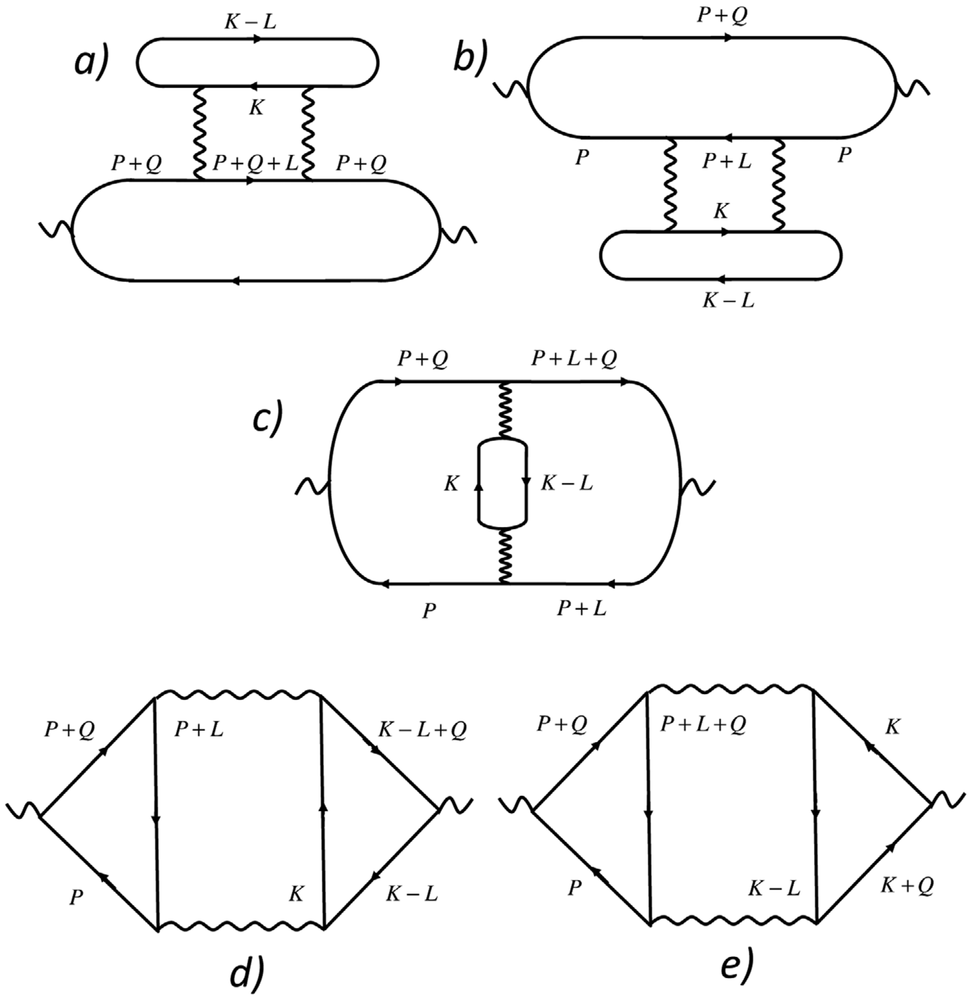
$$= \frac{1}{\omega^2}\sum_{\mathbf{k'pp'}} \bar{W}_{\mathbf{kp}\to\mathbf{k'p'}}\left(\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{p}} - \mathbf{v}_{\mathbf{k'}} - \mathbf{v}_{\mathbf{p'}}\right)\bullet\mathbf{E}$$

$\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{p}} - \mathbf{v}_{\mathbf{k'}} - \mathbf{v}_{\mathbf{p'}} \neq 0$ in any non-Galilean-invariant system

$$\Rightarrow \delta f^{(1)} \propto \frac{E}{\omega^2\tau_{\text{ee}}} \Rightarrow \sigma' = \frac{\omega^2{}_p}{4\pi}\frac{1}{\omega^2\tau_{\text{ee}}}$$

FL: $1/\tau_{\text{ee}} \propto \omega^2 + \left(2\pi T\right)^2$

Same statement in the diagrammatic language



Current-current correlaton function

$$K(\omega) = \sum_{\mathbf{kpk'p'}} \Big(\underbrace{\mathbf{v_k} + \mathbf{v_p} - \mathbf{v_{k'}} - \mathbf{v_{p'}}}_{=0 \text{ in a Gal. inv. system}} \Big)^2 \times \text{bunch of Green's functions}$$

Holstein Ann. Phys. 1964
Riseborough PRB 1983
Gornyi & Mirlin PRB 2004
Maebashi & Fukuyama J. Phys. Soc. Japan 1997, 1998
Chubukov & DM Rep. Prog. Phys. 2017

Selection of diagrams: large N or long-range interaction

Warning

$$\sigma'(\omega)=\frac{\Omega_p^2}{4\pi\omega^2\tau_{ee}}$$ looks like an expansion of the Drude formula

$$\sigma_D'(\omega)=\frac{\Omega_p^2}{4\pi}\mathrm{Re}\frac{1}{1/\tau_{ee}-i\omega}$$ for $\omega\tau_{ee}\gg 1$

Q: Do we have the Drude formula at all ω?

A: No.

2-band model (again) $\Rightarrow$ "superconductor at finite ω"

without intra-band momentum relaxation:

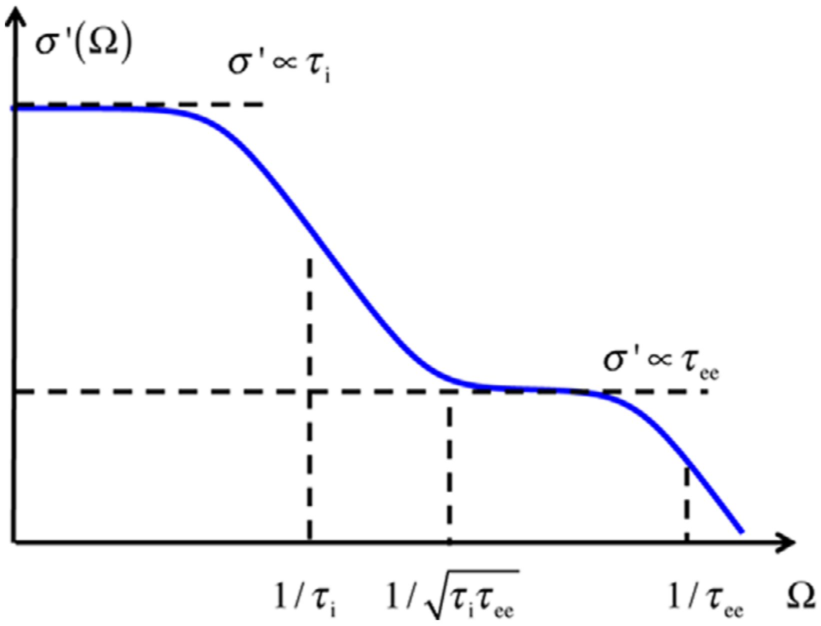
$$\sigma(\omega)=\frac{\Omega_p^2-\Omega_0^2}{4\pi}\frac{1}{\tau_{ee}^{-1}-i\omega}+i\frac{\Omega_0^2}{4\pi(\omega+i\delta)}$$

$$\Omega_p^2=4\pi e^2\left(\frac{n_1}{m_1}+\frac{n_2}{m_2}\right);\ \Omega_0^2=4\pi\frac{\left(e_1n_1+e_2n_2\right)^2}{n_1m_1+n_2m_2}$$

$$\sigma'(\omega)=\frac{\Omega_0^2}{4}\delta(\omega)+\text{regular part}$$

If the metal is not compensated ($\Omega_0\neq 0$), $\sigma'(0)=\infty$

2-band model with intra-band momentum relaxation

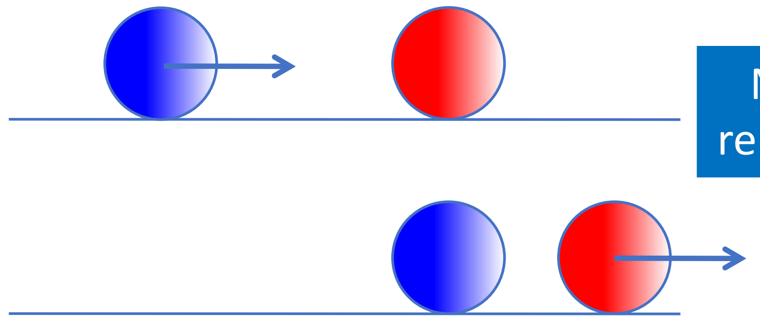


2D: slow relaxation of current

Gurzhi, Kopeliovich, Rutkevich, JETP 1980
Maebashi & Fukuyama, JPSJ 1997
Rosch &Howell, Adv. Phys. 2005
Yudson, Chubukov, DM, PRL 2011
Briskot et al. PRB 2015
Ledwith, Guo, Levitov 2017; 2019

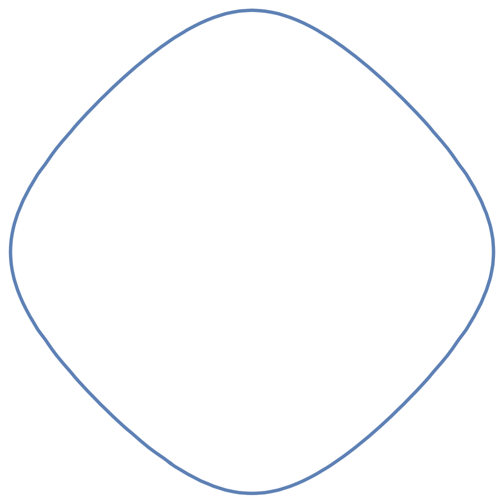
Hidden integrability of motion on a 2D **convex** Fermi surface

1D:

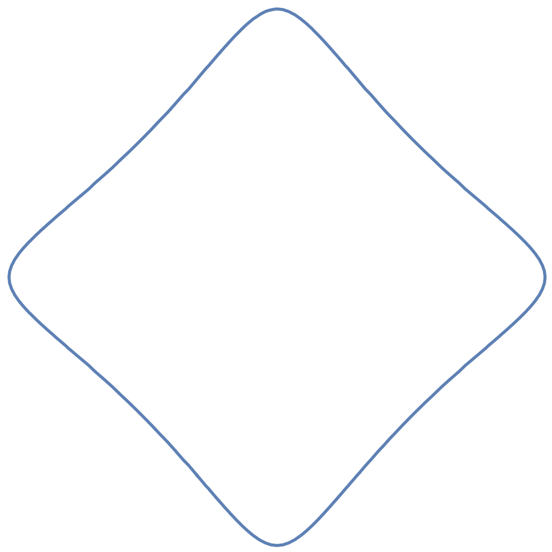


No energy and momentum relaxation (in binary collisions)

“Integrability” in 2D:

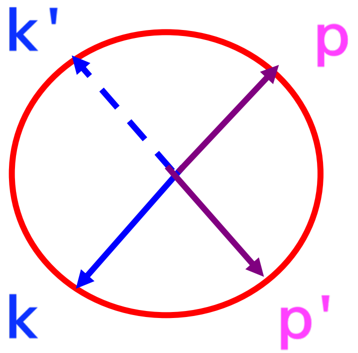


$$\rho = \rho_{\text{imp}} + 0 \times T^2 + BT^4$$

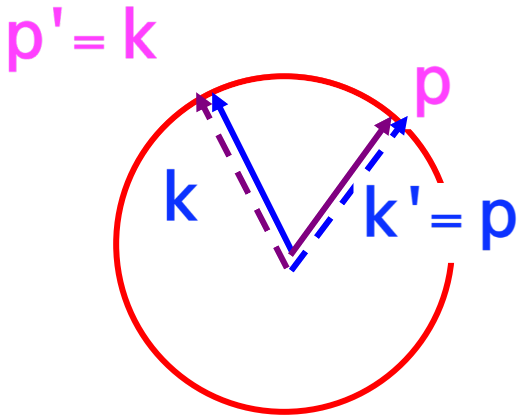


$$\rho = \rho_{\text{imp}} + AT^2$$

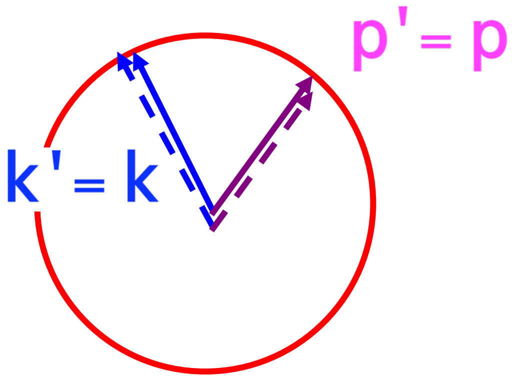
Solution on a circle: only three possible scattering processes



Cooper channel



swapping



forward scattering

$$\Delta \mathbf{v} = \underbrace{\mathbf{v}_{\mathbf{k}} + \mathbf{v}_{\mathbf{p}}}_{0} \underbrace{- \mathbf{v}_{\mathbf{k}'} - \mathbf{v}_{\mathbf{p}'}}_{0}$$

$$\Delta \mathbf{v} = \underbrace{\mathbf{v}_{\mathbf{k}} - \mathbf{v}_{\mathbf{p}'}}_{0} + \underbrace{\mathbf{v}_{\mathbf{p}} - \mathbf{v}_{\mathbf{k}'}}_{0}$$

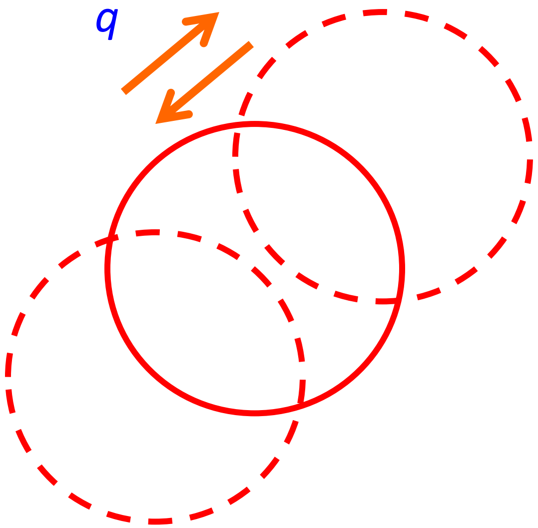
$$\Delta \mathbf{v} = \underbrace{\mathbf{v}_{\mathbf{k}} - \mathbf{v}_{\mathbf{k}'}}_{0} + \underbrace{\mathbf{v}_{\mathbf{p}} - \mathbf{v}_{\mathbf{p}'}}_{0}$$

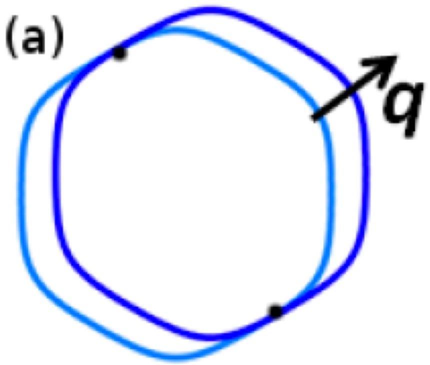
Neither of these processes relaxes current→

$$\rho = \rho_{\text{imp}} + 0 \times T^2$$

Why so few choices?
A circle is convex: at most two self-intersection points

Same is true for any convex FS

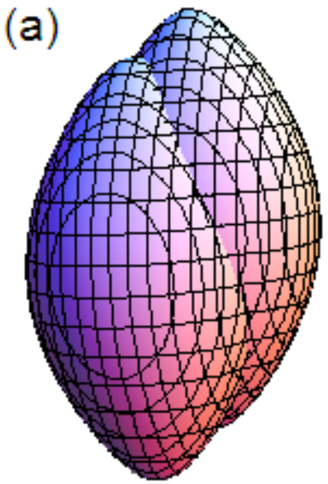


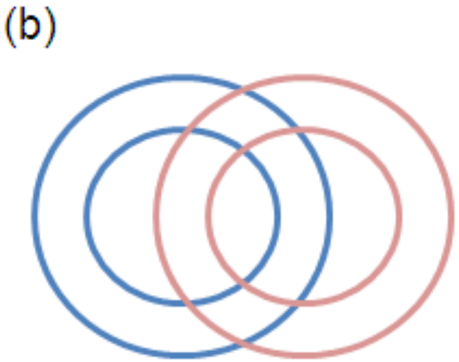


$$\rho = \rho_{\text{imp}} + 0 \times T^2 + BT^4 + \ldots$$

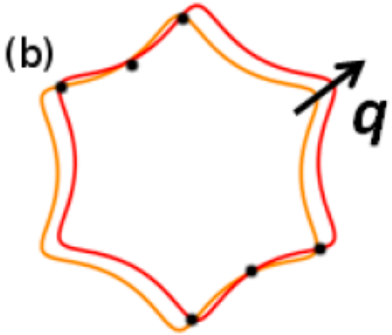
Integrability is broken if the Fermi surface has more than two self-intersection points

3D: any non-quadratic FS





multiband



2D: concave FS

The puzzle of the T^2 resistivity in quantum paraelectrics

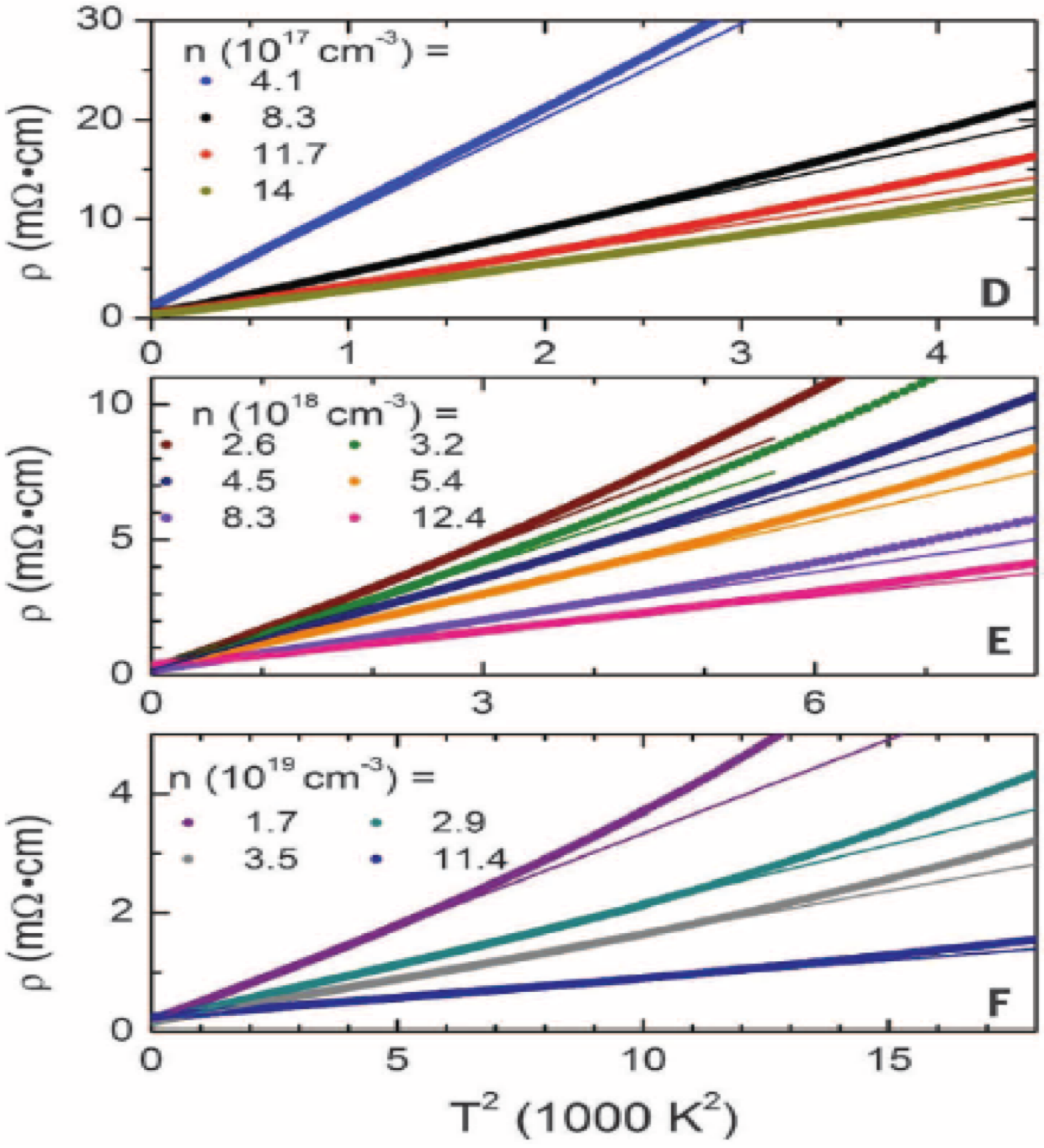
$SrTiO_3$, $KTaO_3$, PbTe...

Scalable T^2 resistivity in a small single-component Fermi surface

Xiao Lin, Benoît Fauqué, Kamran Behnia*

Science **349, 945 (2015)**

Quantum paraelectric:
Almost a ferroelectric which did not make it due to zero-point motion.
Strong fluctuations of the electric dipole moment.



Doped $SrTiO_3$ (STO)

$$\rho = \rho_0 + AT^2$$

What's so surprizing?

Almost everything

1) n~10^{17} cm^{-3}: Single-pocket, tiny, slightly non-spherical Fermi surface at the BZ center: no umklapps, no compensation (e^- only)

2) Quantum paraelectric: $\varepsilon(\omega=0)\approx 25{,}000 \Rightarrow r_s \approx 0.01 \Rightarrow \omega_p \sim 1\ \mathrm{K} \ll E_F \sim 10\ \mathrm{K}$

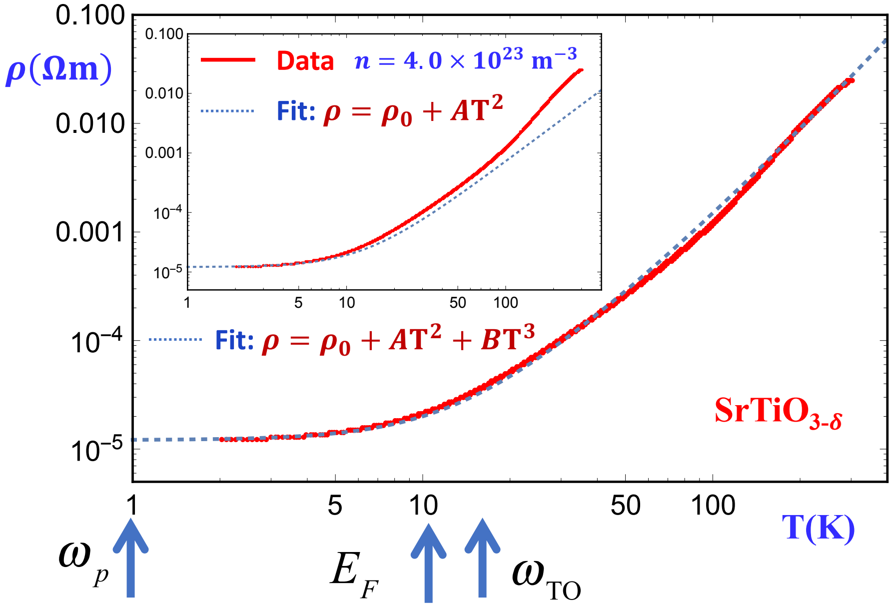
3) T^2 goes through plasma frequency, Fermi energy ,TO phonon frequency (~20 K)

4) Weak e-e interaction: $\rho_{\mathrm{theor}} \sim 10^{-5}\times\rho_{\mathrm{exp}}$ (Maiti, Yudson, DM unpublished)

5) Even single-particle T^2 sets in only at $T < T_{\mathrm{FL}} \sim \omega_p \sim 1\ \mathrm{K} \ll E_F \sim 10\ \mathrm{K}$

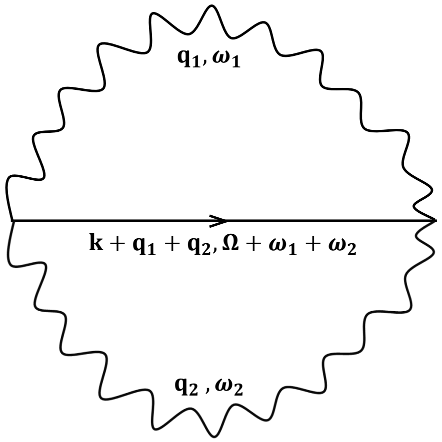
6) τ depends on n only weakly

Abhishek Kumar
UF



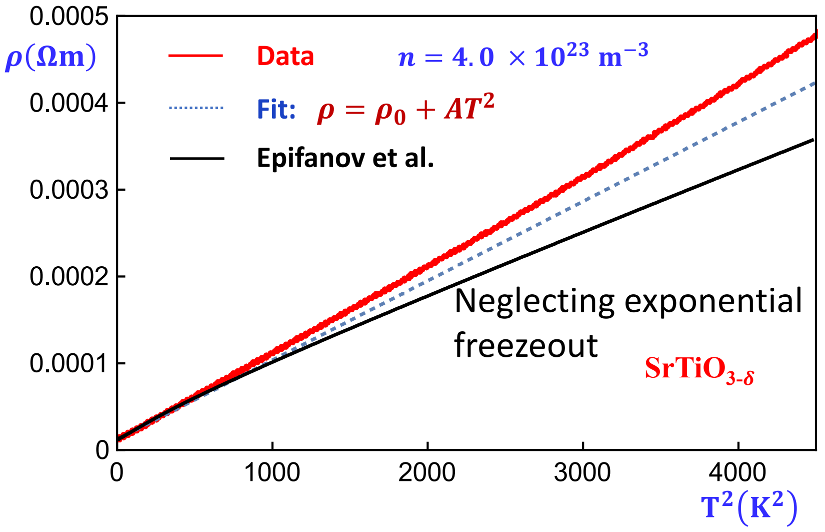
Electron-two TO phonon scattering

Epifanov, A. P. Levanyuk, G. M. Levanyuk, Ferroelectrics ***35****, 199 (1981)*
Sov. Phys. Solid State Phys. ***23****, 391 (1981)*



Transverse polarization: $\mathbf{q}\cdot\mathbf{P}=0$ but $\mathbf{P}^2 \neq 0$

For $T > \omega_{\mathrm{TO}}$, $N_{\mathrm{ph}} \approx \dfrac{T}{\omega_{\mathrm{TO}}}$, $\rho \propto N_{\mathrm{ph}}^2 \propto T^2$



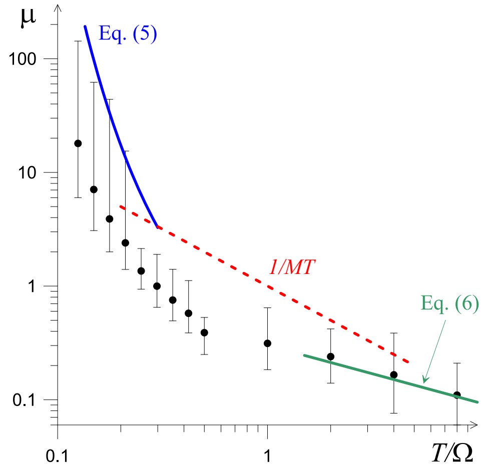
A. Kumar & DM, unpublished

1) Good news: quasi-T^2, abs. value of ρ~correct, no marked variation at $T \sim E_F$, no dependence on n

2) Bad news: exponential freezeout for $T < \omega_{\mathrm{TO}} \sim 20$ K is not observed

Speculations:

1) Ferroelectric domain boundaries in STO (*Scott et al. PRL 2011*) effective suppression of ω_{TO} in transport
2) Recent diagMonte Carlo: Froelich polaron: delayed onset of the exponential freezeout at intermediate & strong coupling

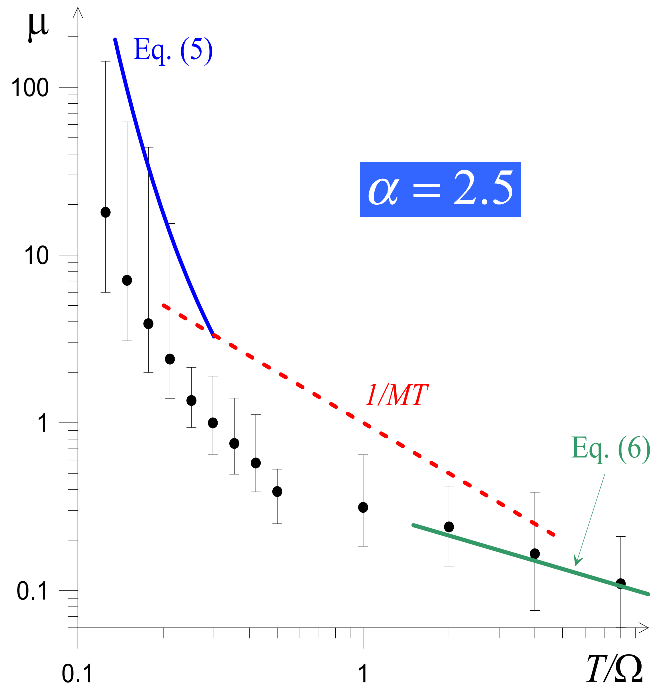


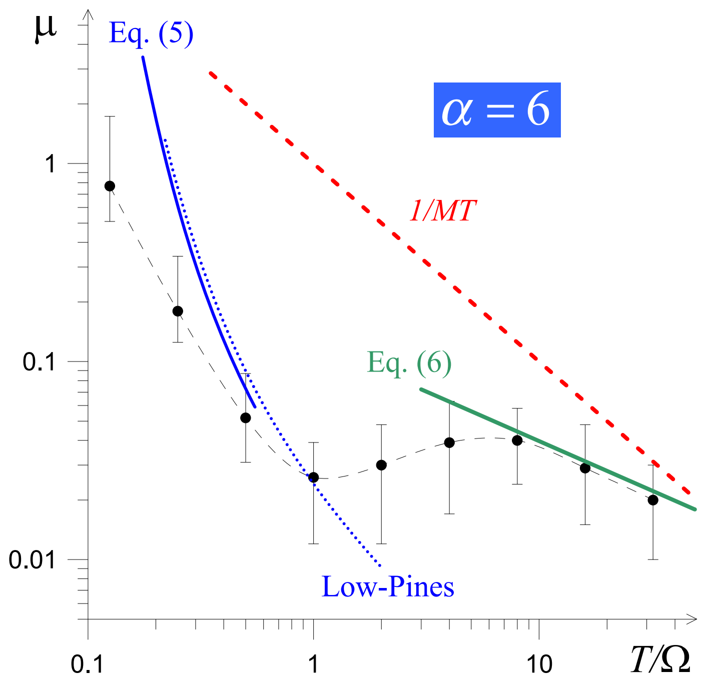
Mischenko et al. 2018 (unpublished)

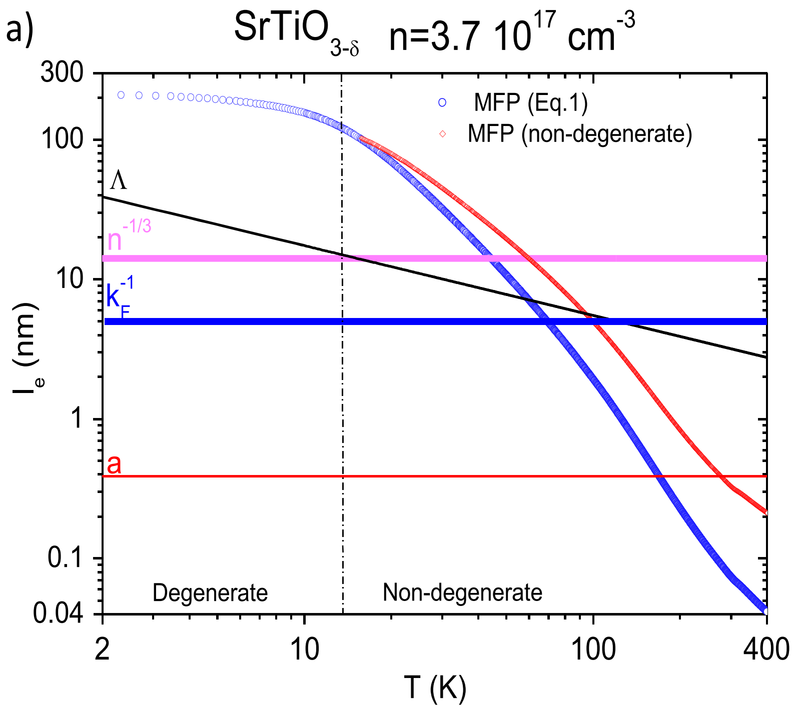
Violation of the Mott-Ioffe-Regel limit in a thermal metal

Mott-Ioffe-Regel limit (continuum): $\ell > \lambda$

Thermal metal: $\lambda = 1/\sqrt{2MT} \Rightarrow 1/\tau < T \quad (k_B T/\hbar) \Rightarrow \mu = e\tau / M > e/MT$







diag Monte Carlo Mischenko et al. 2018 (unpublished)

Lin et al. npj QM 2017

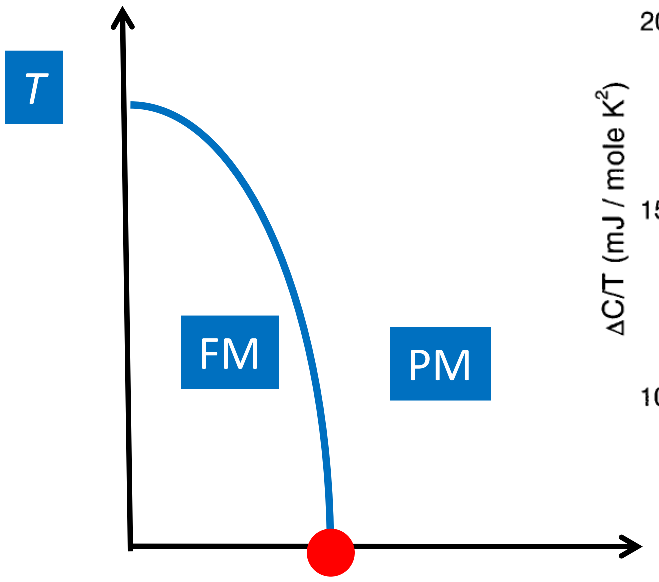
Outline

2. Coherent transport of incoherent quasiparticles

2a. Electric and thermal transport near a ferromagnetic quantum critical point

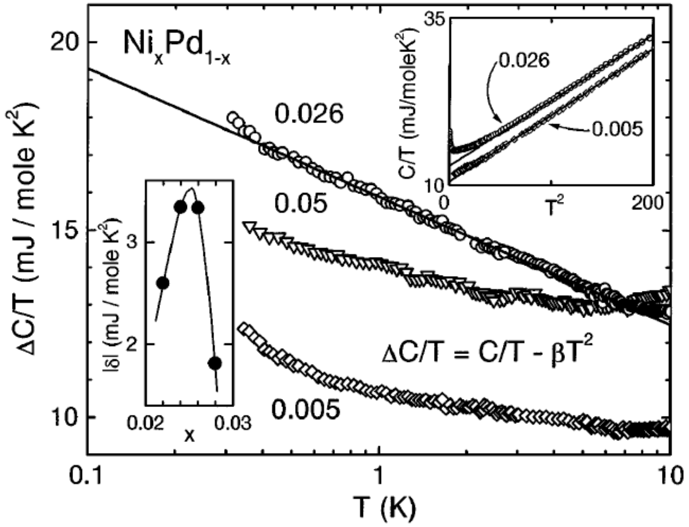
One possible sources of the NFL behavior: Quantum Critical Point

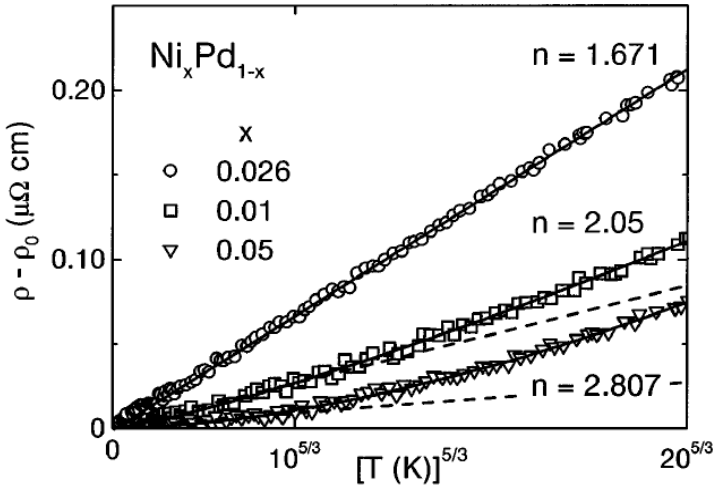
Simplest case: Pomeranchuk ($q=0$) critical point between
a “paramagnet” and a uniformly ordered phase
Hertz-Millis criticality



Quantum Critical point

control parameter



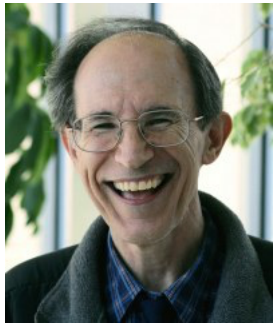


M. Nicklas, M. Brando, G. Knebel, F. Mayr, W. Trinkl, and A. Loidl

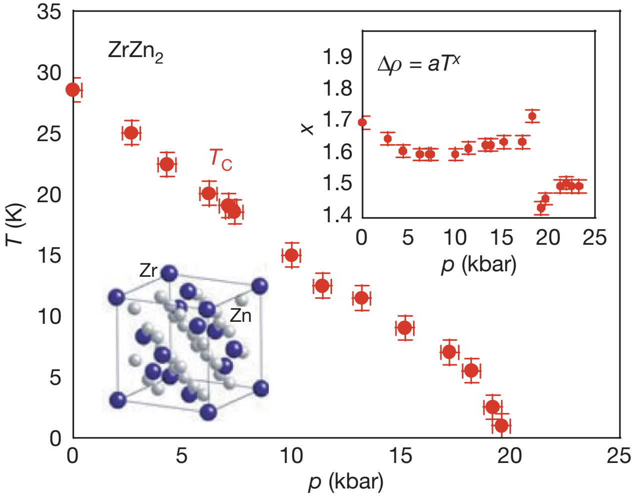
PRL **82,** 4268 (1999)

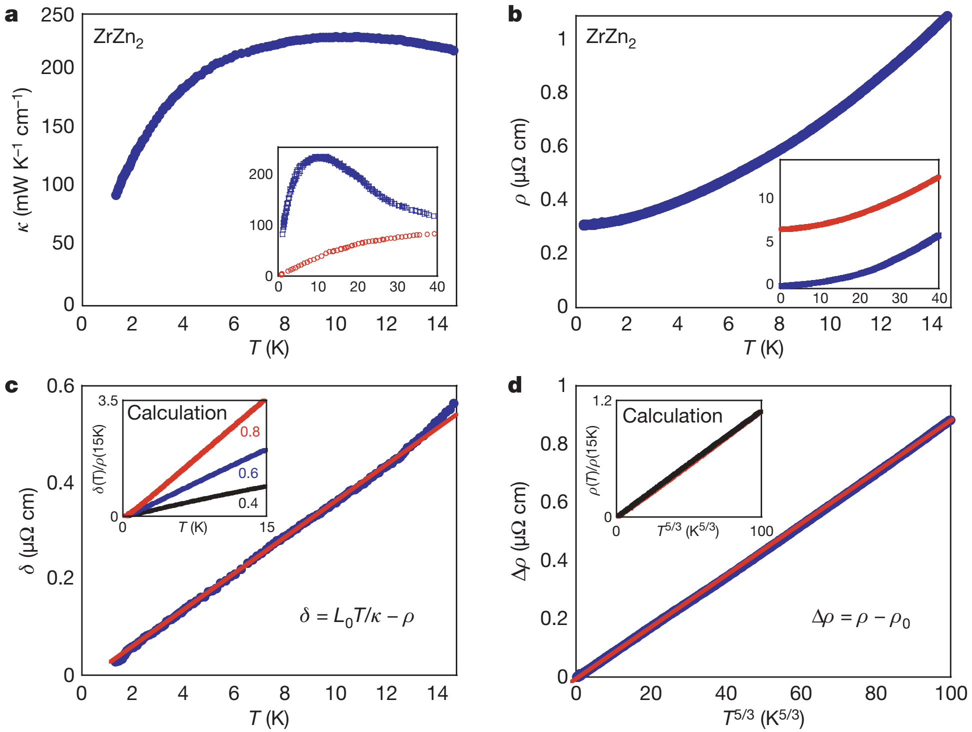
Pd: Compensated metal

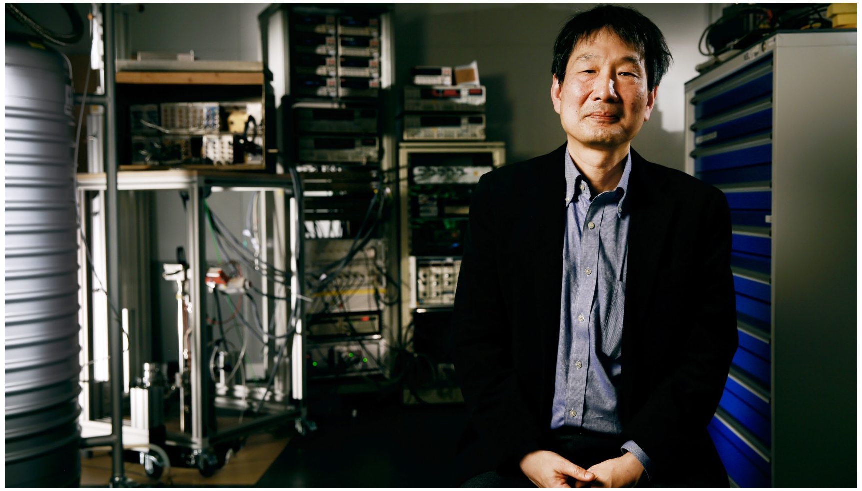
Weak ferromagnet $ZrZn_2$: Hertz-Millis criticality



R. P. Smith[1], M. Sutherland[1], G. G. Lonzarich[1], S. S. Saxena[1], N. Kimura[2], S. Takashima[3], M. Nohara[3] & H. Takagi[3,4] Nature 455, 1220 (2008)







Compensated metal (closed/open FS)

z=3 (Hertz-Millis criticality)

$$\chi^{-1}(q,\Omega_m) \propto q^2 + \xi^{-2} + \gamma |\Omega_m| / q$$

$$\Sigma(\omega_m) \sim \int_0^{\omega_m} d\Omega \int dq_\parallel q_\parallel^{D-2} \chi(q,\Omega_m) \sim \begin{cases} \omega_m^{2/3}, \ \text{2D} \\ \omega_m \ln \omega_m, \ \text{3D} \end{cases}$$

$$m^* / m = 1/Z = 1 - \partial \Sigma'(\omega) / \partial \omega \sim \begin{cases} \omega^{-1/3}, \ \text{2D} \\ |\ln \omega|, \ \text{3D} \end{cases}$$

$$\tau_{\text{sp}}^{-1}(T) = -2\Sigma''(\omega \sim T) \sim \begin{cases} T^{2/3}, \ \text{2D} \\ T, \ \text{3D} \end{cases}$$

$$\tau^{-1}_{\text{tr}}(T) \sim \tau_{\text{sp}}^{-1}(T)(\bar{q} / k_F)^2 \sim \begin{cases} T^{4/3}, \ \text{2D} \\ T^{5/3}, \ \text{3D} \end{cases}$$

$C / T \propto m^* (\omega \sim T) \propto |\ln T|$, 3D $\vee$

Thermal conductivity: $\kappa \sim T v_F^2 \tau_{\text{sp}}$

Thermal resistivity: $w = T / \kappa \propto 1 / \tau_{\text{sp}} \propto T$, 3D $\vee$

Charge resistivity: $\rho \propto 1/\tau_{\text{tr}} \propto T^{5/3}$, 3D $\vee$

compensated metal

$$\sigma = \frac{ne^2\tau}{m}$$

Which mass: bare or renormalized?

Examples:

i) Froelich polaron (*Langreth & Kadanoff, Phys. Rev. 1964*)

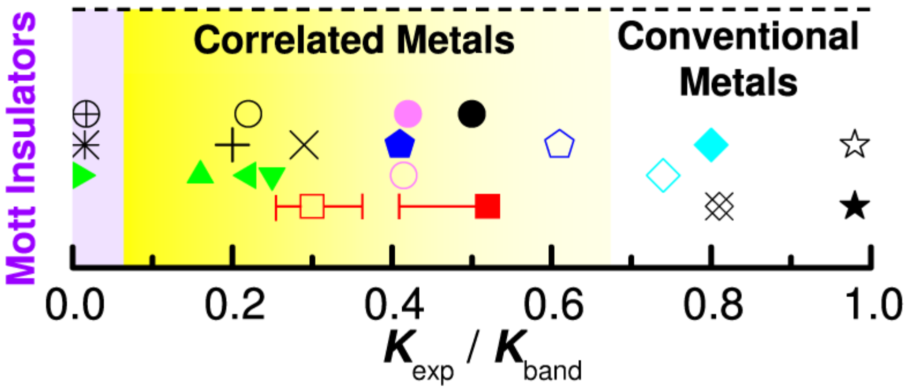
$T < \Omega_{LO}$: renormalized mass

$T > \Omega_{LO}$: bare mass

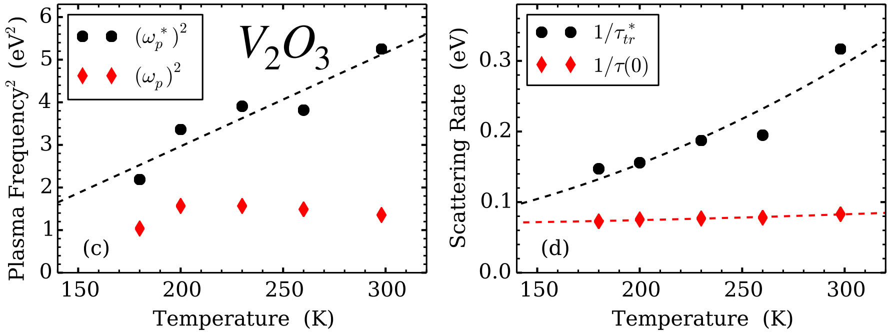
ii) Fermi liquid with impurities (*Langer Phys. Rev. 1960,1961; Michaeli & Finkel'stein PRB 2009*)

both m and τ_{imp} are renormalized. If $\Sigma_{FL} = \Sigma_{FL}(\varepsilon) \Rightarrow \tau_{imp}/m =$ bare

iii)

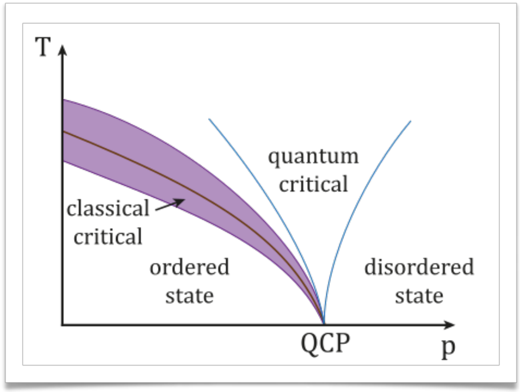


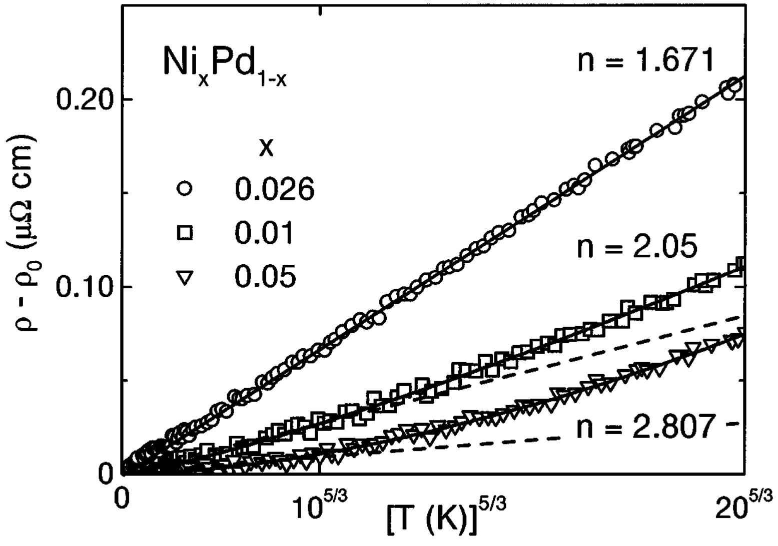
Basov et al. RMP 2011

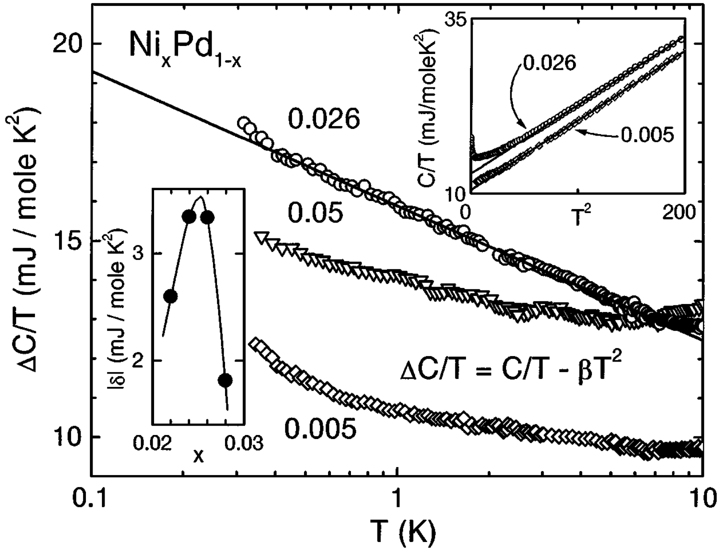


Deng et al. PRL 2014

The “which mass?” question is especially relevant near QCP







Nicklas et al. PRL 1999

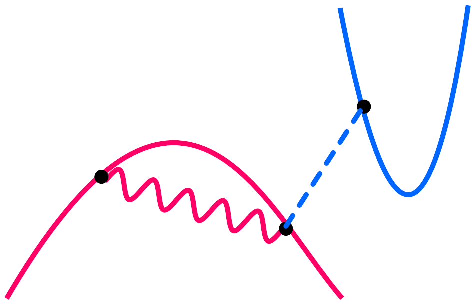
$z = 3$ criticality in 3D : $1/\tau_{\mathrm{tr}} \propto T^{5/3}; m(T) \propto |\ln T|$

$$\rho(T) \propto m^*(T)/\tau_{\mathrm{tr}}(T) \propto T^{5/3}\,|\ln T|\,; w \propto T \ln T?$$

$z = 3$ criticality in 2D : $1/\tau_{\mathrm{tr}} \propto T^{4/3}; m(T) \propto T^{-1/3}$

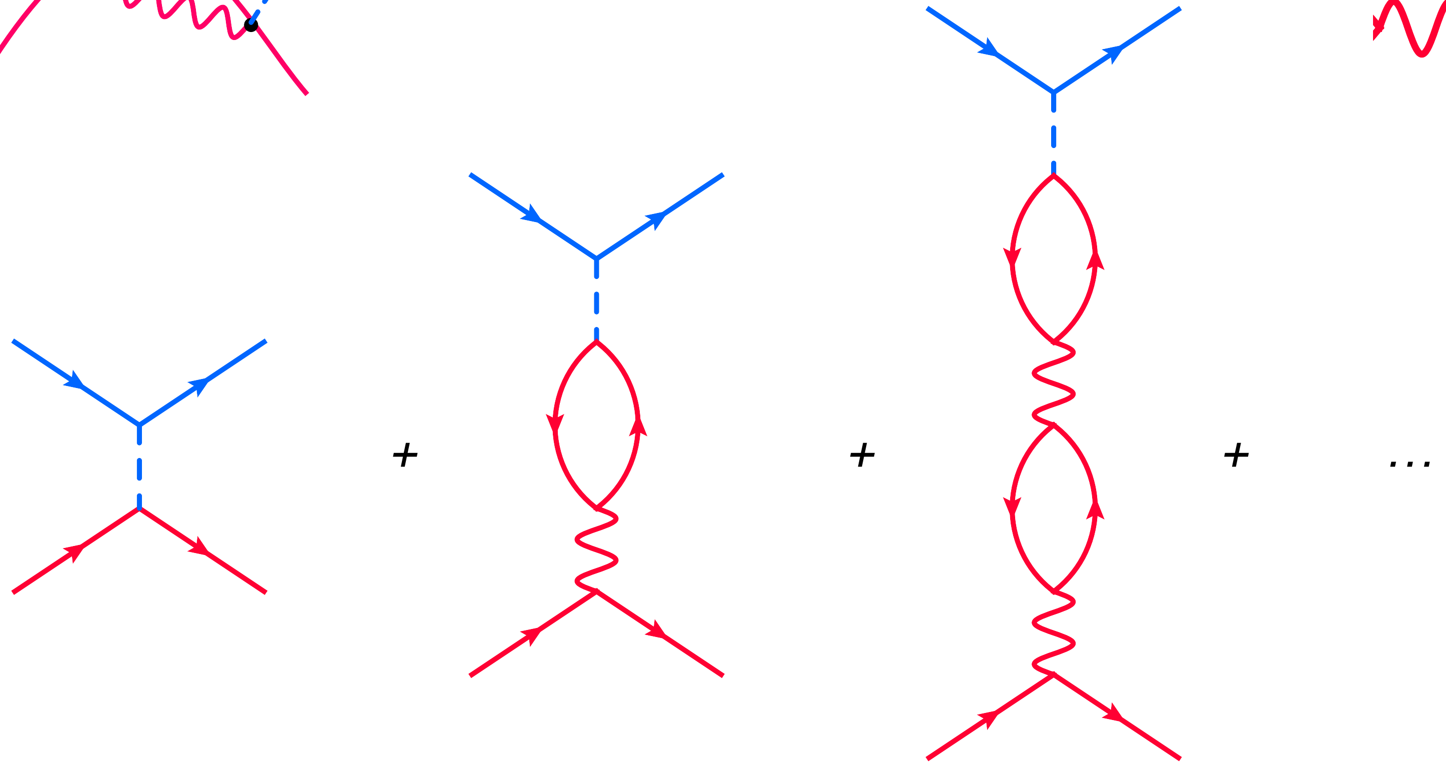
$$\rho(T) \propto m^*(T)/\tau_{\mathrm{tr}}(T) \propto T; w \propto T^{1/3}?$$

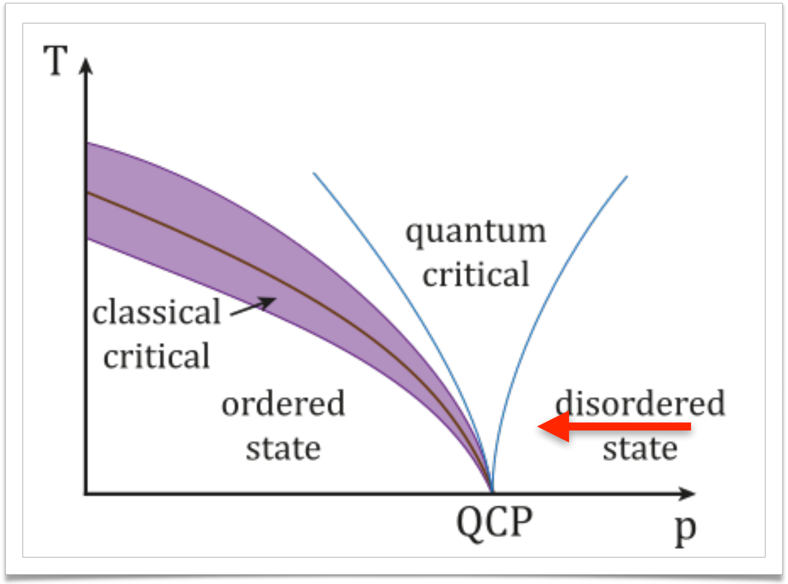
Model: quantum criticality in a compensated metal



Pomeranchuk (e.g. ferromagnetic) cirticality in one (heavier) band

$$\sim \frac{1}{q^2 + \xi^{-2} - i\gamma\Omega/q}$$





Model is controlled for a (strongly) renormalized FL away from QCP

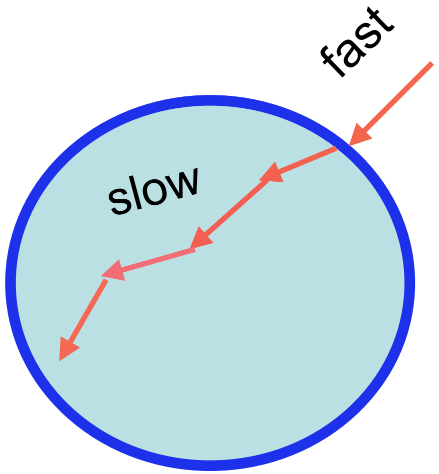
$a_0 \ll \xi < \infty$; Crossover to QC regime: $\xi^{-1} \to T^{1/3}$

Fermi-liquid Kinetic Equation

$$-e(\mathbf{E}\cdot\hat{p})v^*_{F,1}n'_{F,1} = -I^{12}[f_1, f_2]$$
$$-e(\mathbf{E}\cdot\hat{k})v^*_{F,2}n'_{F,2} = -I^{21}[f_1, f_2]$$

- **Exact solution of coupled integral eqs (Li & DM PRB 2018)**
- **Long-range critical fluctuations: forward scattering**
- **Approximate solution**

NB: *dc* KE contains only renormalized masses



$$\delta f_i(\mathbf{p}) = (\mathbf{E}\cdot\hat{\mathbf{p}})\phi_i\left(\xi_{i,\mathbf{p}}\right)$$
$$\phi_i \approx \text{const}$$
$$2\times 2 \text{ linear system for } \phi_{1,2} \rightarrow$$

$$\rho(T) = \frac{1}{ne^2}\frac{1}{48\pi^2}T^2(m_1^* m_2^*)^2\int\frac{dq q^{D-1}}{p_F^{6-D}}W(q) \propto T^2(m_1^* m_2^*)^2\xi^{4-D}$$
$$W(q) = \chi^2(q,\Omega=0) \propto (q^2+\xi^{-2})^{-2}$$
$$\Rightarrow m^*_{1,2}(\xi) \propto \xi^{3-D}$$

$$z=3,\ D=2:\ \xi\rightarrow T^{-1/3},\ m^*_{1,2}\rightarrow T^{-1/3}$$
$$\rho \propto T^{5/3}\times \ln^4 T (?)$$

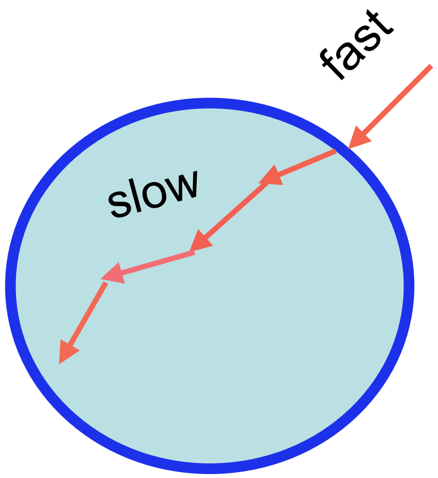
$$z=3,\ D=2:\ \xi\rightarrow T^{-1/3},\ m^*_{1,2}\rightarrow T^{-1/3}$$
$$\rho \propto T^{4/3}\times T^{-4/3} = \text{const}(?)$$

Fermi-liquid Kinetic Equation

$$-e(\mathbf{E}\cdot\hat{p})v^*_{F,1}n'_{F,1} = -I^{12}[f_1,f_2]$$
$$-e(\mathbf{E}\cdot\hat{k})v^*_{F,2}n'_{F,2} = -I^{21}[f_1,f_2]$$

- **Exact solution of coupled integral eqs (Li & DM PRB 2018)**
- **Long-range critical fluctuations: forward scattering**
- **Approximate solution**

NB: *dc* KE contains only renormalized masses

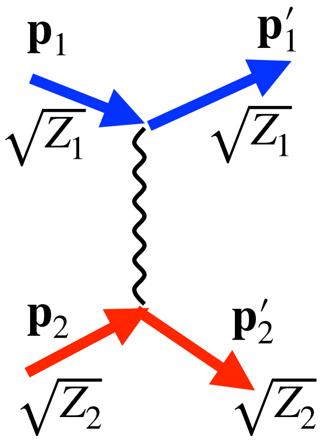


$$\delta f_i(\mathbf{p}) = (\mathbf{E}\cdot\hat{\mathbf{p}})\phi_i\left(\xi_{i,\mathbf{p}}\right)$$
$$\phi_i \approx \text{const}$$
$$2\times 2 \text{ linear system for } \phi_{1,2} \rightarrow$$

$$\rho(T) = \frac{1}{ne^2}\frac{1}{48\pi^2}T^2(m_1^*m_2^*)^2\int\frac{dqq^{D-1}}{p_F^{6-D}}W(q) \propto T^2(m_1^*m_2^*)^2\xi^{4-D}$$
$$W(q) = \chi^2(q,\Omega=0) \propto (q^2+\xi^{-2})^{-2}$$

$$\text{If } \xi \rightarrow T^{-1/3} \text{ @ fixed mass: } \rho \propto T^{\frac{D+2}{3}}$$

$$\text{3D: If } \xi \rightarrow T^{-1/3} \text{ @ } m_{1,2} \propto |\ln T|: \rho \propto T^{\frac{5}{3}}\ln^4 T \ (?) \qquad \text{2D: If } \xi \rightarrow T^{-1/3} \text{ @ } m_{1,2} \propto T^{-1/3}: \rho \propto T^{\frac{4}{3}}T^{-\frac{4}{3}} = \text{const} \ (?)$$



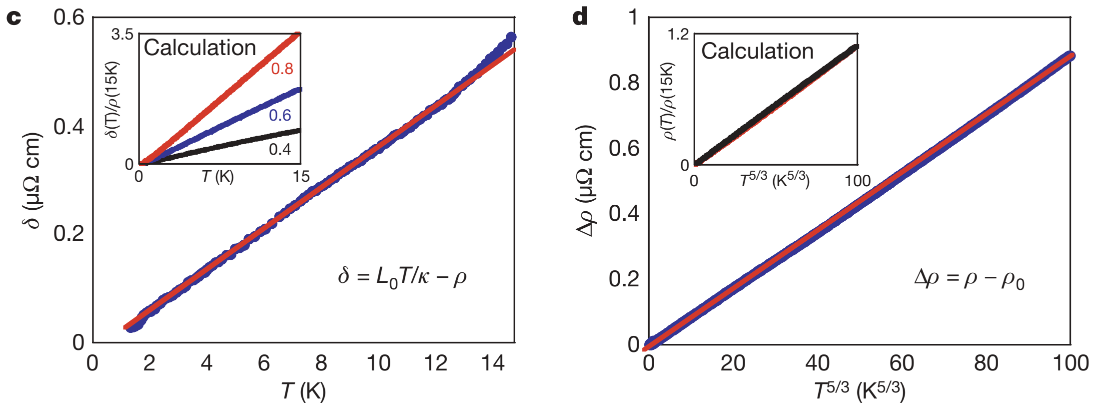
$$W = 2\pi Z_1^2 Z_2^2 \,|\chi(q, \Omega = 0)|^2$$

Local theory : $Z_i m_i^* / m_i = 1$

$$\rho(T) = \frac{1}{ne^2} C_D T^2 (\underbrace{Z_1 m_1^*}_{=m_1} \; \underbrace{Z_2 m_2^*}_{=m_2})^2 \int \frac{dq q^{D-1}}{p_F^{6-D}} |\Gamma|^2 \propto T^2 \xi^{4-D} \to T^{\frac{D+2}{3}}$$

Thermal conductivity: finite already in the single-band case

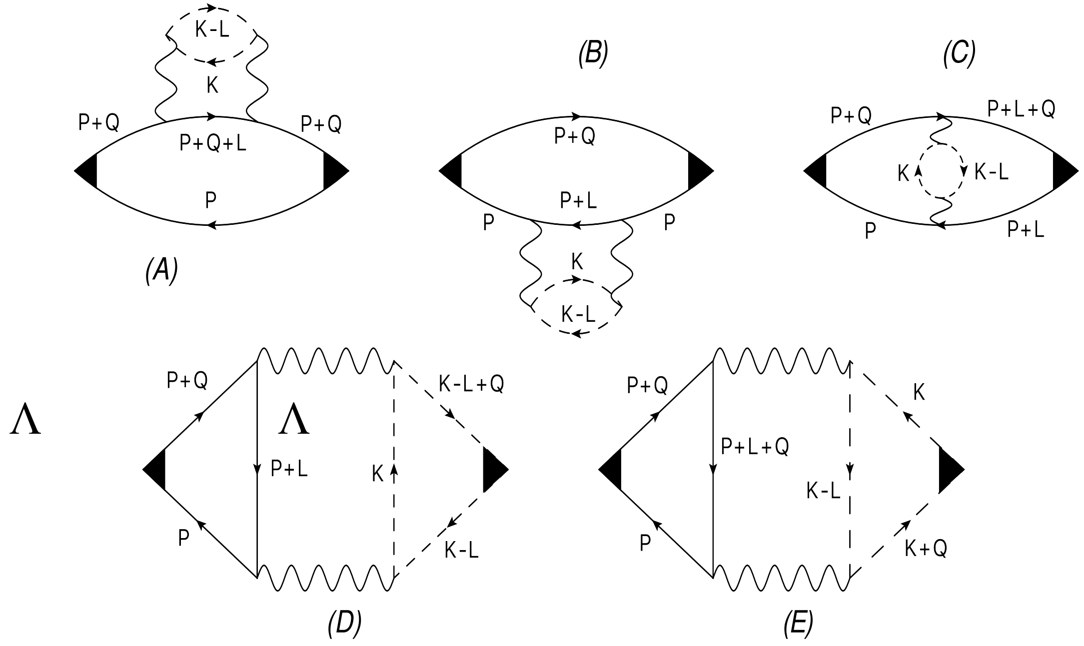
Thermal resistivity: $w(T) \equiv \dfrac{T}{\kappa(T)} \propto T^2 \underbrace{(Zm^*)}_{=m}{}^4 \displaystyle\int dq q^{D-3} |\Gamma|^2 \propto T^2 \xi^{6-D} \to T^{\frac{D}{3}}$



$ZnZr_2$

Smith et al.
Nature 2008

Which mass enters the optical conductivity?



$$T = 0$$

Riseborough, PRB 1983
Kim, Furusaki, Wen, Lee PRB 1994
Eberlein, Mandel, Sachdev PRB 2014
Chubukov & DM PRB 2017
Li & DM unpublished

$$G_\alpha(\mathbf{p}, \varepsilon) = \frac{1}{\frac{\varepsilon}{Z_\alpha} - \xi_{\mathbf{p},\alpha} + \frac{i}{2\tau_{\text{sp}}}}, \alpha = 1{,}2$$

$$\to \sigma'_{\text{A+B}}(\omega) \propto \frac{Z^2}{\omega^2 \tau_{\text{sp}}(\omega)}$$

other diagrams : $\tau_{\text{sp}} \to \tau_{\text{tr}}$

$$\sigma(\omega) \propto \sum_{i=1,2} \Lambda_i^2 Z_i^2 \frac{1}{\omega^2 \tau_{\text{tr}}(\omega)} \propto \sum_{i=1,2} \Lambda_i^2 Z_i^2 \xi^{4-D}$$

@QCP : $\xi \to \omega^{-1/3} \Rightarrow \sigma(\omega) \propto \sum_{i=1,2} \Lambda_i^2 Z_i^2 \omega^{-\frac{4-D}{3}}$

$$Z_{1,2} \propto \xi^{-(3-D)} \to \omega^{\frac{3-D}{3}}$$

Ward identity (charge conservation) : $\Lambda_i Z_i = 1$

$$\sigma(\omega) \propto \omega^{-\frac{4-D}{3}}$$

Is it always true that the bare mass enters the conductivity?

No.

Counter-examples:

1) Froelich polaron for $T \ll \Omega_0$

2) SDW criticality (Chubukov & DM 2017)